Blue

22

Extend and Succeed

Brain Growth - Senior Phase

$$(3x-2)(x+5)=0$$

$$3x-2=0 \quad \text{or} \quad x+5=0$$

$$+2 \quad +2 \qquad\qquad -5 \quad -5$$

$$3x=2 \qquad\qquad x=-5$$

$$\frac{3x}{3}=\frac{2}{3}$$

$$x=\frac{2}{3}$$

Quadratic Equations

and the discriminant

Quadratic Equations

O1 I can solve quadratic equations by factorisation

1. Factorise fully

(a) $3x^2 + 6x$ (b) $15t + 10t^2$ (c) $14z^2 - 7z$

(d) $6x^2 + 8x$ (e) $25r^2 - 10r$ (f) $3v - 12v^2$

(g) $16x^2 - 9$ (h) $4p^2 - 81$ (i) $9x^2 - 16$

(j) $18 - 2x^2$ (k) $3x^2 - 300$ (l) $12k^2 - 75$

2. Use your answers to question 1 to help solve the following Quadratic Equations.

(a) $3x^2 + 6x = 0$ (b) $15t + 10t^2 = 0$

(c) $14z^2 - 7z = 0$ (d) $6x^2 + 8x = 0$

(e) $25r^2 - 10r = 0$ (f) $3v - 12v^2 = 0$

(g) $16x^2 - 9 = 0$ (h) $4p^2 - 81 = 0$

(i) $9x^2 - 16 = 0$ (j) $18 - 2x^2 = 0$

(k) $3x^2 - 300 = 0$ (l) $12k^2 - 75 = 0$

3. Factorise fully

(a) $x^2 - x - 6$ (b) $x^2 - 3x - 10$ (c) $x^2 - 6x + 8$

(d) $x^2 - x - 12$ (e) $x^2 - 6x - 7$ (f) $x^2 - 8x + 15$

(g) $2x^2 - 5x - 3$ (h) $3x^2 + x - 2$ (i) $8x^2 + 14x + 3$

(j) $10x^2 + 19x - 15$ (k) $3x^2 + 8x + 4$ (l) $4x^2 + 5x - 6$

4. Use your answers to question 3 to help solve the following Quadratic Equations.

(a) $x^2 - x - 6 = 0$

(b) $x^2 - 3x - 10 = 0$

(c) $x^2 - 6x + 8 = 0$

(d) $x^2 - x - 12 = 0$

(e) $x^2 - 6x - 7 = 0$

(f) $x^2 - 8x - 15 = 0$

(g) $2x^2 = 5x + 3$

(h) $3x^2 + x = 2$

(i) $8x^2 = -14x - 3$

(j) $10x^2 + 19x = 15$

(k) $3x^2 + 8x + 4 = 0$

(l) $0 = 6 - 5x - 4x^2$

5. Factorise fully

(a) $3x^2 + 6x - 24$

(b) $15x^2 + 5x$

(c) $2x^2 - 32$

(d) $5x^3 - 45x$

(e) $18x^2 - 6x - 12$

(f) $12x^2 + 8x$

(g) $10x^2 + 25x - 15$

(h) $6x^3 + 30x^2 + 36x$

(i) $7x^2 - 28$

(j) $2x^2 - 10x + 12$

(k) $3x^3 + 21x^2 - 54x$

(l) $6x^3 - 63x$

6. You can use your answers to question 5 to help solve the following Quadratic (and Cubic) Equations.

(a) $3x^2 + 6x - 24 = 0$

(b) $15x^2 + 5x = 0$

(c) $2x^2 - 32 = 0$

(d) $5x^3 - 45x = 0$

(e) $18x^2 - 6x = 12$

(f) $12x^2 + 8x = 0$

(g) $10x^2 = 15 - 25x$

(h) $6x^3 + 30x^2 + 36x = 0$

(i) $7x^2 = 28$

(j) $2x^2 - 10x + 12 = 0$

(k) $3x^3 + 21x^2 = 54x$

(l) $6x^3 = 63x$

7. Solve the following (disguised) quadratic equations

(a) $(x-2)^2 = 16$ (b) $3x^2 = 10x - 8$

(c) $3(x+1)^2 = 3$ (d) $2x(4-x) + 3(4-x) = 0$

8. Solve the following (disguised) quadratic equations

(a) $x = \frac{6}{x-1}$ (b) $x - 3 = \frac{5x-14}{x+2}$

(c) $x + 1 = \frac{1}{x+1}$ (d) $x + 8 = \frac{9}{x+8}$

9. Solve the following (disguised as quadratic) equations

(a) $x^4 - 13x^2 + 36 = 0$ (b) $x^4 - 15x^2 - 16 = 0$

(c) $x^4 - 5x^2 + 6 = 0$ (d) $x^6 + 7x^3 = 8$

10. Solve the following without multiplying out the brackets

(a) $(2x+1)^2 - (x+2)^2 = 0$ (b) $(3x-2)^2 - (2x+1)^2 = 0$

(c) $(4x+1)^2 - (x-3)^2 = 0$ (d) $(3x-1)^2 - (x+7)^2 = 0$

11. (a) Factorise $[f(x)]^2 - [g(x)]^2$

(b) Given $f(x) = 5x - 3$ and $g(x) = 4x - 5$

Solve $[f(x)]^2 - [g(x)]^2 = 0$ without multiplying out brackets.

Cumulative Exercises - Linear Algebra

1. Solve these equations

(a) $8 - 2(x - 1) = 4$ (b) $x - 2(x + 1) = 8$

(c) $\frac{2}{x} + 1 = 6$ (d) $2x - \frac{(3x-1)}{4} = 4$

2. Solve these Inequalities

(a) $3 - 2x < 4x - 3$ (b) $3x > 2 - (x - 6)$

(c) $-2(1 + 3x) \leq 3x + 1$ (f) $\frac{x}{4} - \frac{1}{2} < 5$

3. Change the subject of $p = \frac{2(m-4)}{3}$ to m.

4. Change the subject of $L = \frac{1}{2}(h - t)$ to h.

5. Change the subject of $T = \frac{\sqrt{a}}{k}$ to a.

6. Change the subject of $W = BH^2$ to H.

7. Solve algebraically the system of equations

(a) $2x + 3y = 8$
$5x - 2y = 1$

(b) $3x + 5y = 7$
$4x + 3y = 2$

8. Given $f(x) = 9 - x$ and $g(x) = 5x$

State the range of value for which $f(x) > g(x)$.

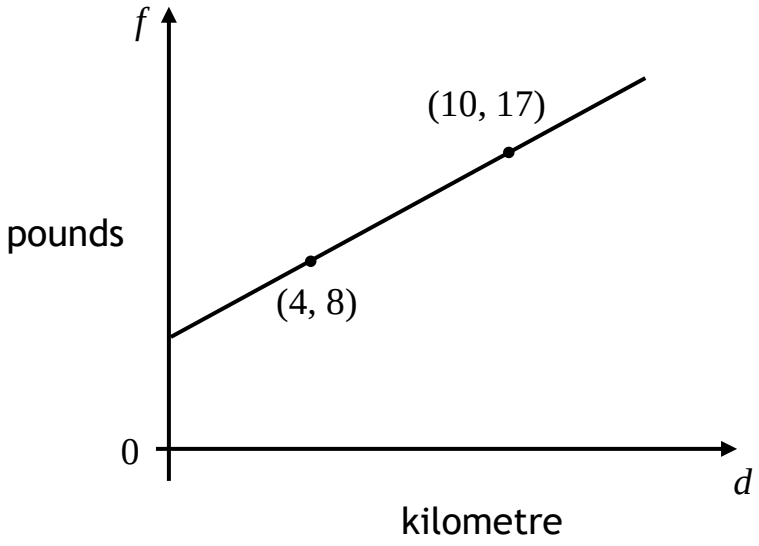
9. The equations below represent different straight lines.

For each straight line state the gradient and the coordinates of the y-intercept.

(a) $y = 5x - 3$ (b) $y = 3 - x$ (c) $2x + y = 7$

(d) $x + 4y = -3$ (e) $2x - y = 7$ (f) $3x - 2y - 8 = 0$

10. The graph shows the fare, f pounds for a journey of d kilometres.



The graph shows that a 4-kilometre journey will cost £8 and a 10-kilometre journey will cost £17.

Find the equation of the straight line.

O2 I can solve Quadratic Equations using the Quadratic Formula

1. Solve these quadratic equations, giving your answers correct to 2 significant figures.

(a) $x^2 + 3x + 1 = 0$

(b) $x^2 + 7x + 2 = 0$

(c) $x^2 - 7x + 11 = 0$

(d) $x^2 + x - 3 = 0$

(e) $2x^2 + 5x + 1 = 0$

(f) $3x^2 - 5x - 1 = 0$

(g) $4x^2 - 13x + 2 = 0$

(h) $2x^2 + 7x + 2 = 0$

2. Solve these quadratic equations (some are disguised), giving your answers correct to 2 significant figures.

(a) $1 + 6x - x^2 = 0$

(b) $8 - x - x^2 = 0$

(c) $-2x^2 = 8x - 4$

(d) $6 - x^2 = 9x$

(e) $2 - x^2 = x(x + 1)$

(f) $2(x^2 - 2) = 3$

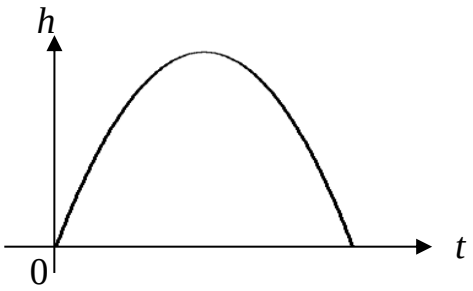
(g) $x - 3 + \frac{1}{x} = 0$

(h) $3 - 4x^2 = 10x$

3. Given $2x^2 - 2x - 1 = 0$, show that $x = \frac{1 \pm \sqrt{3}}{2}$.

Book 22 - #2

1. The diagram below shows the path of a rocket which is fired into the air.



The height, h metres, of the rocket after t seconds is given by

$$h(t) = -2t(t - 14).$$

(a) For how many seconds is the rocket in flight?

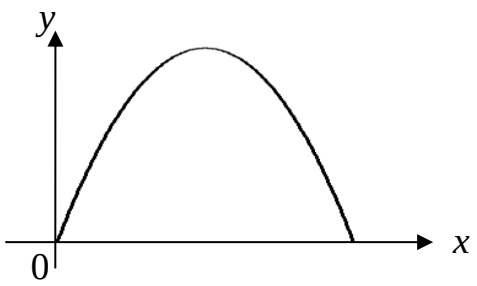
(b) What is the maximum height reached by the rocket?

2. The profit made by a publishing company of a magazine is calculated by the formula

$$y = 4x(140 - x)$$

where y is the profit (in pounds) and x is selling price (in pence) of the magazine.

The graph below represents the profit y against the selling price x.



Find the maximum profit the company can make from the sale of the magazine.

O3 I can use the Discriminant to determine the nature of the roots of a Quadratic Equation.

SQA Guidance

When determining the nature of roots for a quadratic equation, the expected responses are:

- For $b^2 - 4ac > 0$: 'two real and distinct roots'.
- For $b^2 - 4ac = 0$: 'one repeated real root' or 'two equal real roots'.
- For $b^2 - 4ac < 0$: 'no real roots'.

1. Determine the nature of the roots of each quadratic equation using the discriminant.

(a) $x^2 + 8x + 16 = 0$

(b) $x^2 - 3x + 4 = 0$

(c) $2x^2 + 3x - 4 = 0$

(d) $3x^2 - 7x + 2 = 0$

(e) $x^2 - 5x + 3 = 0$

(f) $2x^2 - 4x + 2 = 0$

(g) $4 - 2x - x^2 = 0$

(h) $3 - 3x + 7x^2 = 0$

2. Calculate the discriminant.

(a) $x^2 - 7x - 2 = 0$

(b) $2x^2 - 3x + 2 = 0$

(c) $4 - 7x + x^2 = 0$

(d) $3x + 7 = -5x^2 + 4$

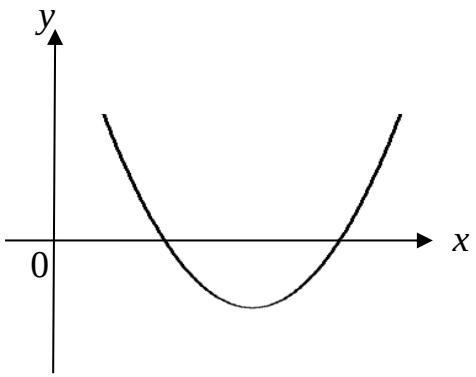
(e) $2 - x^2 = x(x + 1)$

(f) $x - 3 + \frac{1}{x} = 0$

(g) $-x^2 - 4 = 0$

(h) $\frac{4}{x} + x = 5$

3. The graph of $y = f(x)$ is a parabola, part of which shown below.



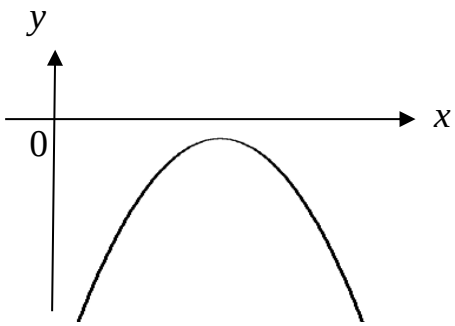
Which of the following statements are true?

(a) The discriminant of $f(x) = 0$ is negative.

(b) When $y = f(x)$ is written as $y = ax^2 + bx + c$ then $a > 0$.

(c) $f(x) = 0$ has two real and distinct roots.

4. The graph of $y = g(x)$ is a parabola, part of which shown below.



Which of the following statements are true?

(a) The discriminant of $g(x) = 0$ is negative.

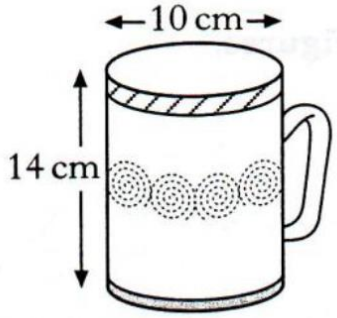
(b) When $y = g(x)$ is written as $y = ax^2 + bx + c$ then $a > 0$.

(c) $g(x) = 0$ has two equal real roots.

Book 22 - #3

DEADLINE

1. A mug is in the shape of a cylinder with (inside) diameter 10 centimetres and height 14 centimetres.

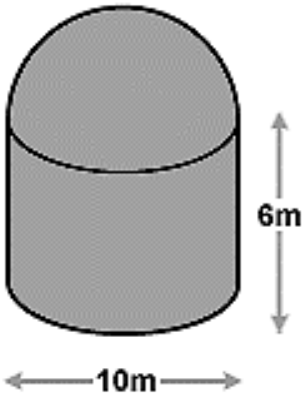


(a) Calculate the volume of the mug.

(b) 600 millilitres of tea are poured into the mug.

Calculate the depth of the tea in the mug.

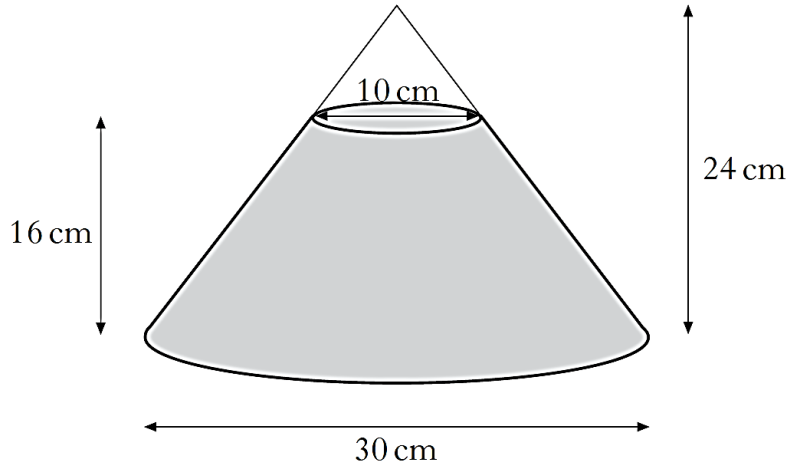
2. A silage tank is made up of a cylinder with a hemisphere on top as shown in the diagram.



(a) Calculate the total volume of the silage tank.

(b) If 300 cubic metres of silage are contained in the cylindical base of the tank, what would be the depth of the silage?

3. A lampshade is in the shape of a truncated cone as shown below.



The cone is 24 centimetres high and has a base of 30 centimtres.

The lampshade has a height of 16 centimetres and measures 10 centimtres across at the top.

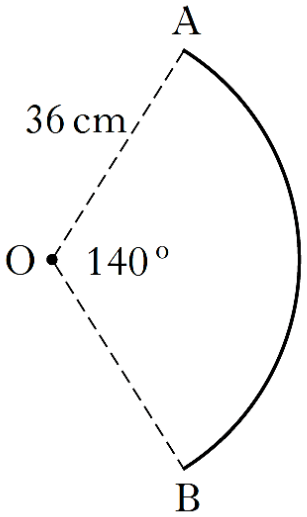
What is the volume of the lampshade?

Give your answer correct to two significant figures.

4. A circle, centre O, has radius 36 centimetres.

Part of this circle is shown.

Angle $A0B = 140°$.

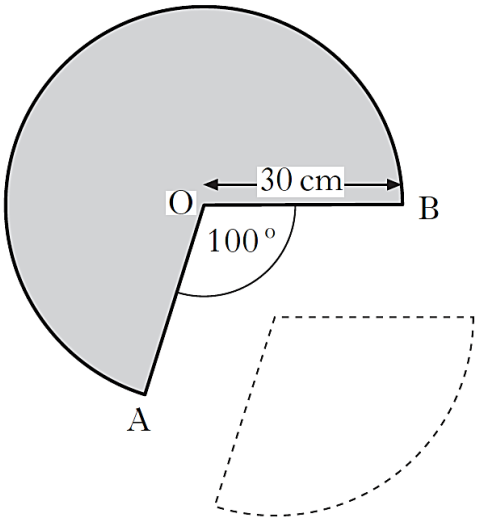


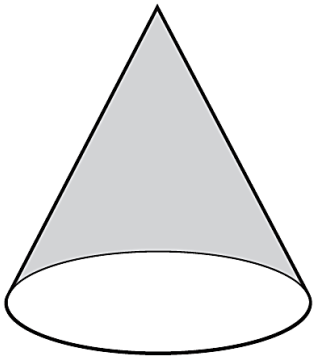
Calculate the area of the sector.

5. A cone is formed from a paper circle with a sector removed as shown.

The radius of the paper circle is 30 cm.

Angle $A0B = 100°$.





(a) Calculate the area of paper used to make the cone.

(b) Calculate the circumference and radius of the base of the cone.

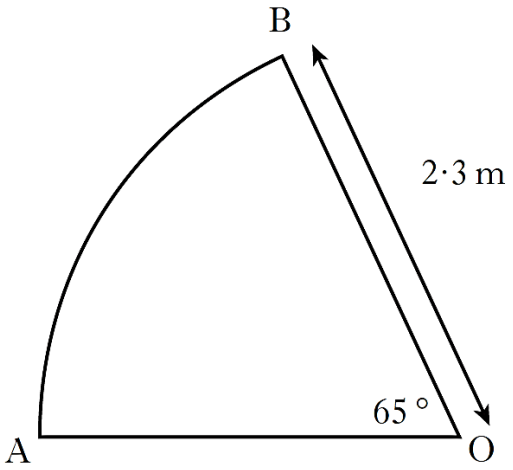
(c) By applying Pythagoras in 3D, calculate the height of the cone.

(d) Calculate the volume of the cone.

6. A circle, centre O, has radius 2·3 metres.

A sector of this circle is shown.

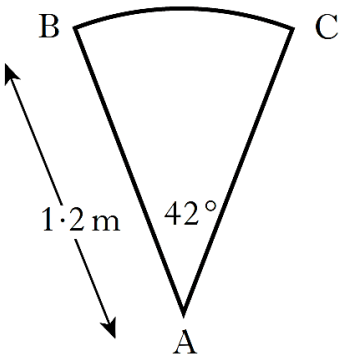
Angle $A0B = 65°$.



Find the length of arc AB.

7. A spiral staircase is being designed.

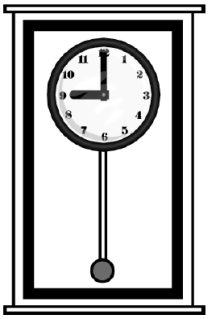
Each step is made from a sector as shown.

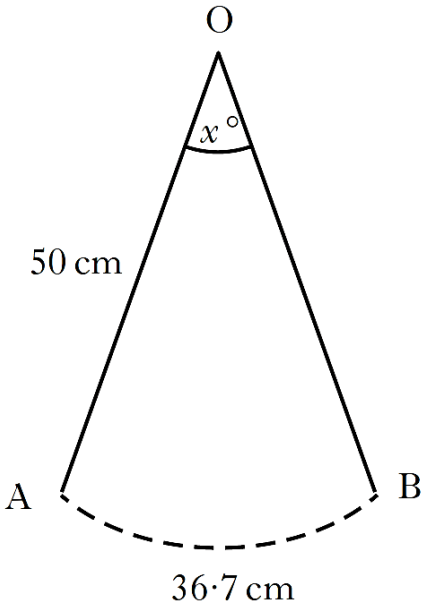


For the staircase to pass safety regulations, the arc BC must be at least 0·9 metres.

Will the staircase pass safety regulations?

8. As the pendulum of a clock swings, its tip moves through an arc of a circle as shown in the diagram.

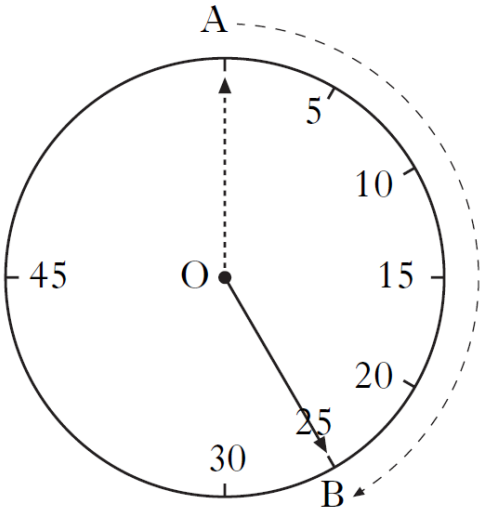




Calculate $x°$, the angle through which the pendulum swings.

9. Contestants in a quiz have 25 seconds to answer a question.

This time is indicated on the clock.

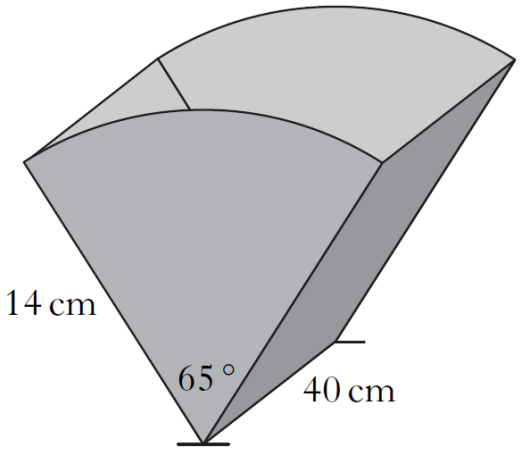


(a) Calculate the size of angle AOB.

(b) The length of arc AB, the distance travelled by the tip of the clock hand, is 120 centimetres.

Calculate the length of the clock hand.

10. The ends of a magazine rack are identical sectors, 40 cm apart, as shown.



What is the volume of the magazine rack?

O4 Problems involving the solution of a Quadratic (or Cubic) Equations

1. A rectangular garden has a length of $(x + 7)$ metres and a breadth of $(x + 3)$ metres.

(a) Show that the area, A square metres, of the garden is given by $A = x^2 + 10x + 21$.

(b) If the area of the garden is 45 square metres, find x.

2. A rectangular garden has a length of $(x + 11)$ metres and a breadth of $(x + 5)$ metres.

(a) Show that the area, A square metres, of the garden is given by $A = x^2 + 16x + 55$.

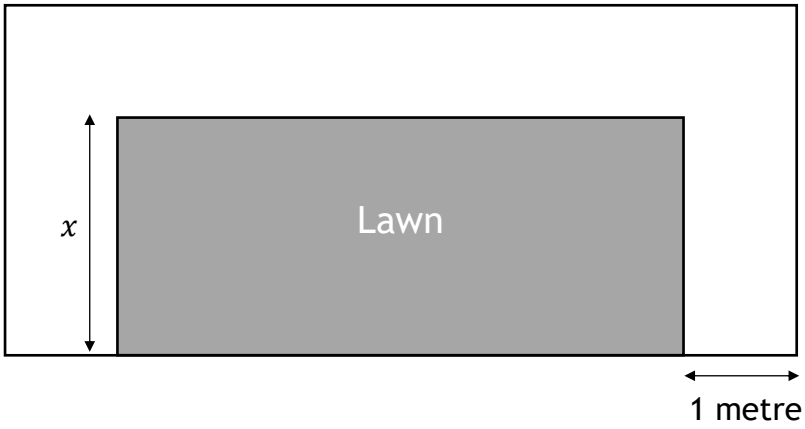
(b) If the area of the garden is 135 square metres, find x.

3. A rectangular garden has a length of $(x + 4)$ metres and a breadth of $(x + 5)$ metres.

(a) Show that the area, A square metres, of the garden is given by $A = x^2 + 9x + 20$.

(b) If the area of the garden is 56 square metres, find x.

4. A rectangular lawn has a path, 1 metre wide, on 3 sides as shown.



The breadth of the lawn is x metres.

The length of the lawn is three times its breadth.

The area of the lawn equals the area of the path.

(a) Show that $3x^2 - 5x - 2 = 0$.

(b) Hence find the length of the lawn.

5. A rectangular wall vent is 30 centimetres long and 20 centimetres wide.

It is to be enlarged by increasing both the length and the width by x centimetres.

(a) Write down expressions for the length and width of the new vent.

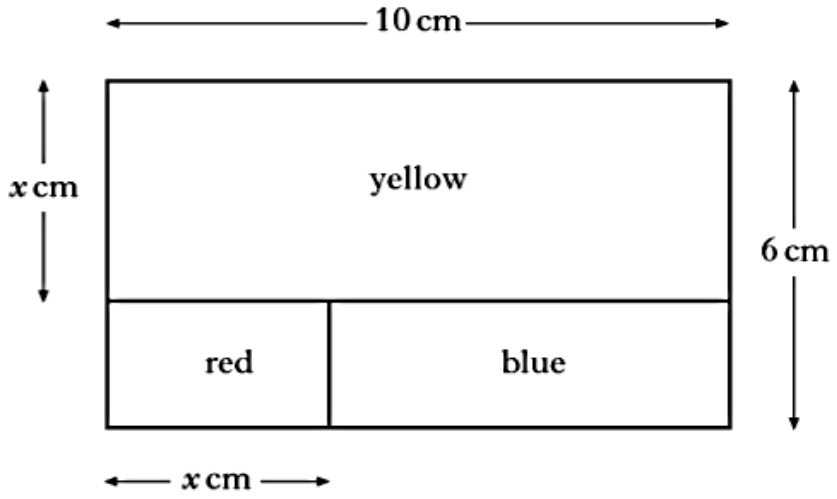
(b) Show that the area, A square centimetres, of the new vent is given by

$$A = x^2 + 50x + 600.$$

(c) The area of the new vent must be at least 40% more than the original area.

Find the minimum dimensions, to the nearest centimetre, of the new vent.

6. A decorator's logo is rectangular and measures 10 centimetres by 6 centimetres.



It consists of three rectangles: one red, one yellow and one blue.

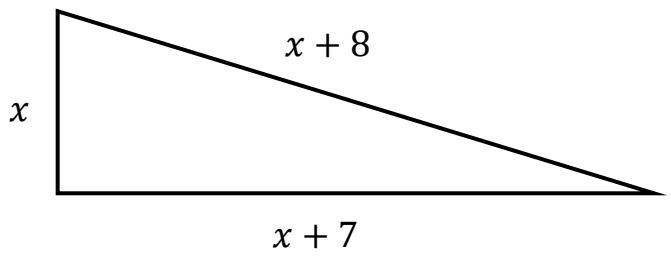
The yellow rectangle measures 10 centimetres by x centimetres.

The width of the red rectangle is x centimetres.

(a) Show that the area, A, of the blue rectangle is given by the expression $A = x^2 - 16x + 60$.

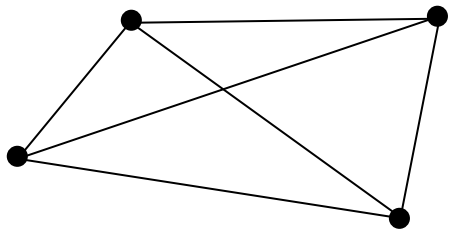
(b) The area of the blue rectangle is equal to $\frac{1}{5}$ of the total area of the logo. Calculate the value of x.

7. A right-angled triangle has dimensions, in centimetres, as shown.



Calculate the value of x.

8. The minimum number of roads joining 4 towns to each other is 6 as shown.



The minimum number of roads, r, joining n towns to each other is given by the formula $r = \frac{1}{2}n(n-1)$.

(a) State the minimum number of roads needed to join 7 towns to each other.

(b) When $r = 55$, show that $n^2 - n - 110 = 0$.

(c) Hence find algebraically the value of n.

9. The number of diagonals, d, in a polygon with n sides is given by the formula $d = \frac{1}{2}n(n-3)$.

(a) (i) A polygon has 65 diagonals.

Show that for this polygon $n^2 - 3n - 130 = 0$

(ii) Hence find the number of sides in this polygon.

(b) A polygon has 20 diagonals.

How many sides does it have?

10. The weight, W kilograms, of a giraffe is related to its age, M months, by the formula

$$W = \frac{1}{4}(M^2 - 4M + 272).$$

At what age will the giraffe weigh 83 kilograms?

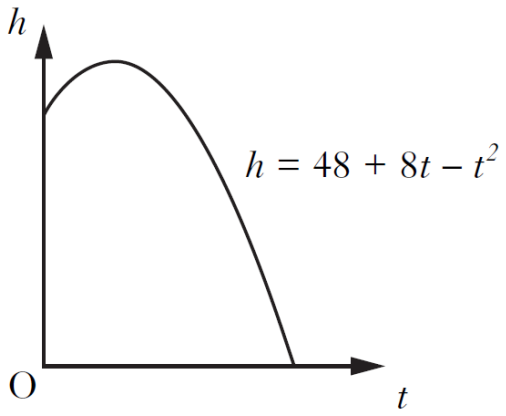
11. The side length of a cube is $2x$ centimtres.

(a) Write an expression for the volume of the cube in cubic centimetres.

(b) Write an expression for the surface are of the cube in square centimetres.

(c) If the expression for the volume is equal to the expression for the surface area, find the length of the cube.

12. The diagram shows that path of a flare after it is fired.



The height, h metres above sea level, of the flare is given by $h = 48 + 8t - t^2$ where t is the number of seconds after firing.

Calculate, algebraically, the time taken for the flare to enter the sea.

O5 Problem Solving involving the discriminant

1. $ax^2 + 4x - 2 = 0$ has two equal and real roots.

Find a.

2. $x^2 + bx + 25 = 0$ has one repeated root.

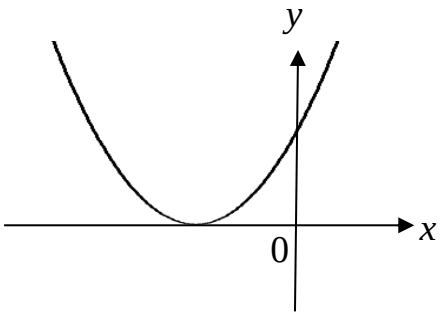
Find 2 values for b.

3. $px^2 + 8x - 2 = 0$ has 2 real and distinct roots.

Set up an inequality in p, and solve for p.

4. $x^2 + x - t = 0$ has no real roots. Solve for t.

5. The graph of $y = g(x)$ is a parabola, part of which shown below.



If $g(x) = x^2 + px + 25 = 0$, use the discriminant to find the value of p.

Book 22 - #4

DEADLINE