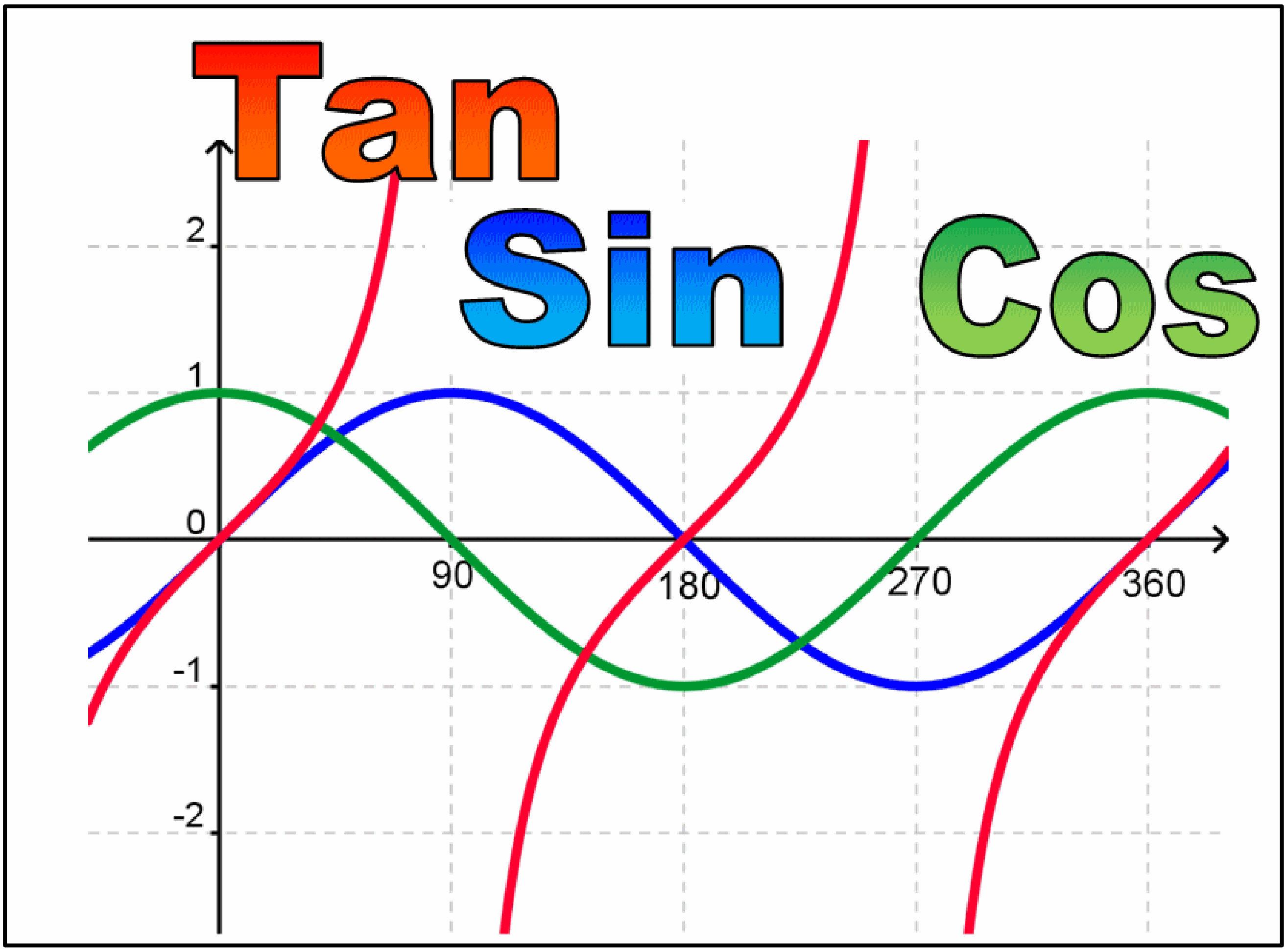
Blue

21

Extend and Succeed

Brain Growth - Senior Phase



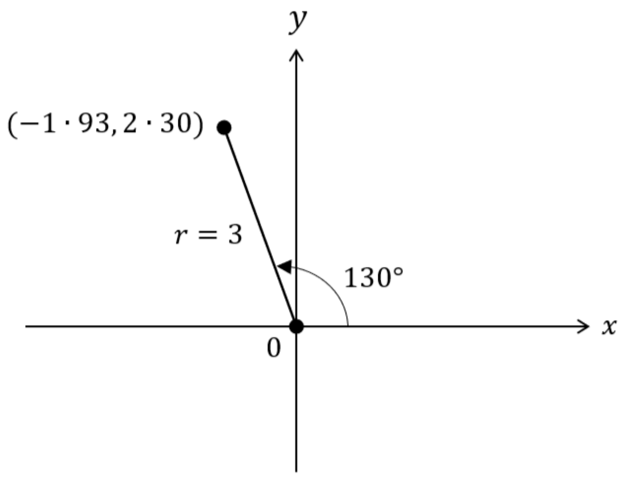
Trigonometry

Graphs and Equations

Trig Graphs

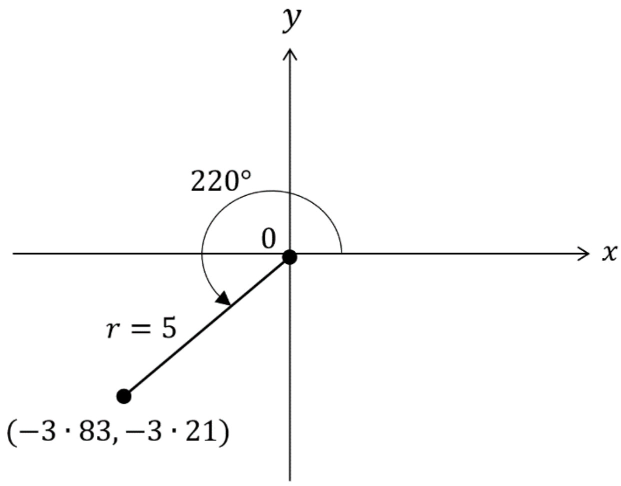
O1 Trig ratios of angles of all sizes

1.



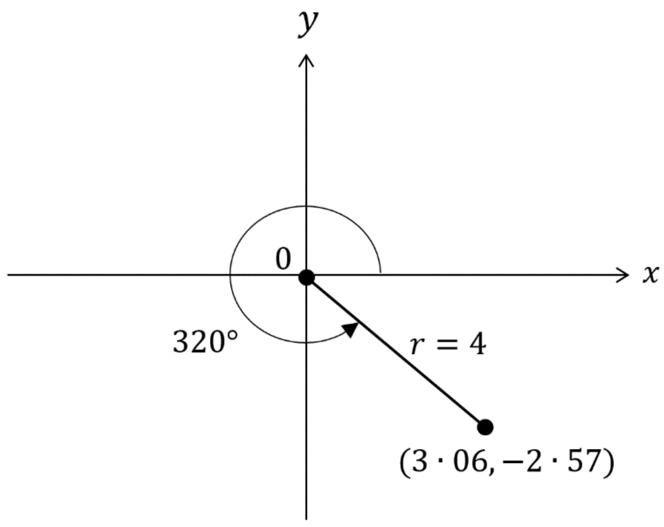
Given the diagram above, find $\sin 130°$, $\cos 130°$ and $\tan 130°$ correct to 2 significant figures.

2.



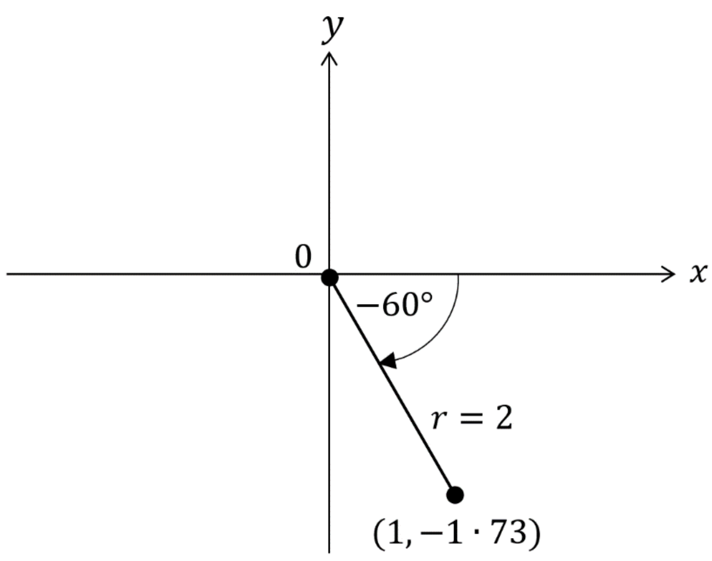
Given the diagram above, find $\sin 220°$, $\cos 220°$ and $\tan 220°$ correct to 2 significant figures.

3.



Given the diagram above, find $\sin 320°$, $\cos 320°$ and $\tan 320°$ correct to 2 significant figures.

4.



Given the diagram above, find $\sin -60°$, $\cos -60°$ and $\tan - 60°$ correct to 2 significant figures.

O2 I can use ICT to plot and analyse Trig Graphs

1. (a) Set the following DOMAIN and step size on Desmos.

DOMAIN $-60 \leq x \leq 420$ STEP SIZE 30

(b) Set the following RANGE.

RANGE $-2 \leq y \leq 2$ STEP SIZE 0.5

(c) Use Desmos to plot $y = \sin \mathrm{x}°$.

(d) Sketch and annotate the graph of $y = \sin \mathrm{x}°$ on worksheet 1.

(e) Write down the amplitude and period of the graph of $y = \sin \mathrm{x}°$.

(f) Write down the coordinates of where your graph of $y = \sin \mathrm{x}°$ cuts the axes and the coordinates of all of the turning points.

2. (a) With the same DOMAIN and RANGE, use Desmos to plot $y = \cos \mathrm{x}°$.

(b) Sketch and annotate the graph of $y = \cos \mathrm{x}°$ on worksheet 1.

(c) Write down the amplitude and period of the graph of $y = \cos \mathrm{x}°$.

(d) Write down the coordinates of where your graph of $y = \cos \mathrm{x}°$ cuts the axes and the coordinates of all of the turning points.

3. (a) Keep the same DOMAIN, but set the following RANGE.

RANGE $-5 \leq y \leq 5$ STEP SIZE 1

(b) Use Desmos to plot $y = \tan \mathrm{x}°$.

(c) Sketch and annotate the graphs of $y = \tan \mathrm{x}°$ on worksheet 1

(a) Investigating the (peak) amplitude

4. Plot the graphs of the following using desmos and look for a connection between the equation and the amplitude.

You will have to adjust the RANGE so that the whole graph fits on the screen.

(a) $y = 2\sin x°$ (b) $y = 5\cos x°$

(c) $y = 10\cos x°$ (d) $y = 5\sin x°$

(e) $y = 3\sin x°$ (f) $y = 15\cos x°$

5. Plot the graphs of the following using desmos and look for a connection between the equation and the amplitude.

(a) $y = 2\sin x° + 1$ (b) $y = 5\cos x° - 1$

(c) $y = 10\cos x° + 3$ (d) $y = 5\sin x° - 2$

(e) $y = 3\sin x° + 1$ (f) $y = 15\cos x° - 5$

6. Predict the amplitude of graphs of each of the following and check your answer by plotting the graph.

(a) $y = 3\sin x° - 4$ (b) $y = 2\cos x° + 1$

(c) $y = \cos x° + 5$ (d) $y = 4\sin x° - 1$

(e) $y = 5\sin x° - 3$ (f) $y = 10\cos x° + 9$

(b) Investigating the vertical translation

7. Plot the graphs of the following using desmos and look for a connection between the equation and vertical translation.

(a) $y = \sin x° + 1$

(b) $y = \cos x° - 1$

(c) $y = \cos x° + 3$

(d) $y = \sin x° - 2$

(e) $y = \sin x° + 1$

(f) $y = \cos x° - 5$

8. Plot the graphs of the following using desmos and look for a connection between the equation and vertical translation.

(a) $y = 2\sin x° + 1$

(b) $y = 5\cos x° - 1$

(c) $y = 10\cos x° + 3$

(d) $y = 5\sin x° - 2$

(e) $y = 3\sin x° + 1$

(f) $y = 15\cos x° - 5$

9. Predict the vertical translation of the graphs of each of the following and check your answer by plotting the graph.

(a) $y = 3\sin x° - 4$

(b) $y = 2\cos x° + 1$

(c) $y = \cos x° + 5$

(d) $y = 4\sin x° - 1$

(e) $y = 5\sin x° - 3$

(f) $y = 10\cos x° + 9$

(c) Investigating the period

10. Plot the graphs of following using desmos and look for a connection between the equation and period.

(a) $y = \sin 2x°$

(b) $y = \cos 3x°$

(c) $y = \cos 2x°$

(d) $y = \sin 4x°$

(e) $y = \sin 3x°$

(f) $y = \cos 4x°$

11. Plot the graphs of following using desmos and look for a connection between the equation and the period.

(a) $y = \sin 2x° + 1$

(b) $y = \cos 3x° - 1$

(c) $y = \cos 2x° + 3$

(d) $y = \sin 4x° - 2$

(e) $y = \sin 3x° + 1$

(f) $y = \cos 4x° - 5$

12. Predict the period of graphs of each of the following and check your answer by plotting the graph.

(a) $y = \sin 3x° - 4$

(b) $y = \cos 2x° + 1$

(c) $y = \cos 5x° + 1$

(d) $y = 4\sin 2x° - 1$

(e) $y = 5\sin 3x° - 3$

(f) $y = 10\cos 4x° + 1$

(d) Investigating the horizontal translation (phase angle)

13. Plot the graphs of following using desmos and look for a connection between the equation and the horizontal shift.

(a) $y = \sin(\mathrm{x} - 30)°$ (b) $y = \cos(\mathrm{x} - 45°)$

(c) $y = \cos(\mathrm{x} - 90)°$ (d) $y = \tan(\mathrm{x} - 45)°$

(e) $y = \tan(\mathrm{x} - 90)°$ (f) $y = \cos(\mathrm{x} - 30)°$

14. Plot the graphs of following using desmos and look for a connection between the equation and the horizontal shift.

(a) $y = \sin(\mathrm{x} + 30)°$ (b) $y = \cos(\mathrm{x} + 45°)$

(c) $y = \tan(\mathrm{x} + 90)°$ (d) $y = \sin(\mathrm{x} + 45)°$

(e) $y = \sin(\mathrm{x} + 90)°$ (f) $y = \tan(\mathrm{x} + 30)°$

15. Predict the horizontal shift of the graphs of each of the following and check your answer by plotting the graph.

(a) $y = 3\sin(\mathrm{x} + 15)°$ (b) $y = \cos(\mathrm{x} + 180°) - 1$

(c) $y = \tan(\mathrm{x} - 30)°$ (d) $y = 3\sin(\mathrm{x} + 45)°$

(e) $y = \sin(\mathrm{x} + 90)° + 2$ (f) $y = \tan(\mathrm{x} - 60)°$

(g) $y = 5\sin(\mathrm{x} - 30)° - 2$ (h) $y = 2\cos(\mathrm{x} + 15°) + 3$

(i) $y = \tan(\mathrm{x} + 90)°$ (j) $y = 4\sin(\mathrm{x} - 45)°$

(e) Investigating reflection through the axes

1. (a) Use Desmos to plot $y = \sin x°$.

   (b) On the same diagram plot the graph of $y = -\sin x°$

   (c) Describe the connection between $y = \sin x°$ and $y = -\sin x°$

2. (a) Use Desmos to plot $y = \cos x°$.

   (b) On the same diagram plot the graph of $y = -\cos x°$

   (c) Describe the connection between $y = \cos x°$ and $y = -\cos x°$

3. (a) Make a conjecture about the connection between the graphs of $y = 3\sin x°$ and $y = -3\sin x°$

   (b) Check your conjecture by plotting the two graphs on the same diagram.

4. (a) Use Desmos to plot $y = \sin x°$.

   (b) On the same diagram plot the graph of $y = \sin(-x)°$

   (c) Describe the connection between $y = \sin x°$ and $y = \sin(-x)°$

5. (a) Use Desmos to plot $y = \cos x°$.

   (b) On the same diagram plot the graph of $y = \cos(-x)°$

   (c) Describe the connection between $y = \cos x°$ and $y = \cos(-x)°$

Cumulative Exercises - Surds and Indices

1. Simplify

(a) $\sqrt{27} + 2\sqrt{3}$ (b) $\sqrt{50} - \sqrt{8}$ (c) $\frac{x^4 \times x^3}{x^5}$

(d) $\frac{y^3 \times y^6}{y^2}$ (e) $4x^2 \times 2x^3$ (f) $2x^3 \times 6x^{-1}$

2. A satellite travels $1 \cdot 22 \times 10^6$ kilometres in a day.
A higher orbit satellite travel 4 times this distance each day.
Calculate the distance the higher orbit satellite travels each day.
Give your answer in scientific notation.

3. Remove the brackets and simplify $b^{\frac{1}{2}}\left(b^{-\frac{1}{2}} + 1\right)$

4. (a) Simplify $a^3 \times \sqrt[3]{a^2}$

(b) Express $p^2(p^4 - p^{-2})$ in its simplest form

5. (a) Simplify $\frac{m^7}{m^3}$

(b) Express $2\sqrt{7} + \sqrt{28} - \sqrt{63}$ as a surd in its simplest form.

6. Express as a fraction in its simplest form.

(a) $\frac{10p^3}{3} \div \frac{p}{6}$ (b) $\frac{3s^3}{t^4} \times \frac{t}{6s}$

7. Simplify $\frac{ab^5}{a^4b^3}$.

O3 I can sketch Trig Graphs.

Use the scales on Blue 21 – Worksheet 2

1. Sketch the graph of $y = -2\sin x°, 0 \leq x \leq 360$.

2. Sketch the graph of $y = 4\cos 2x°, 0 \leq x \leq 360$.

3. Sketch the graph of $y = \sin x° + 2, 0 \leq x \leq 360$.

4. Sketch the graph of $y = 3\cos x° - 1, 0 \leq x \leq 360$.

5. Sketch the graph of $y = 2 - \cos x°, 0 \leq x \leq 360$.

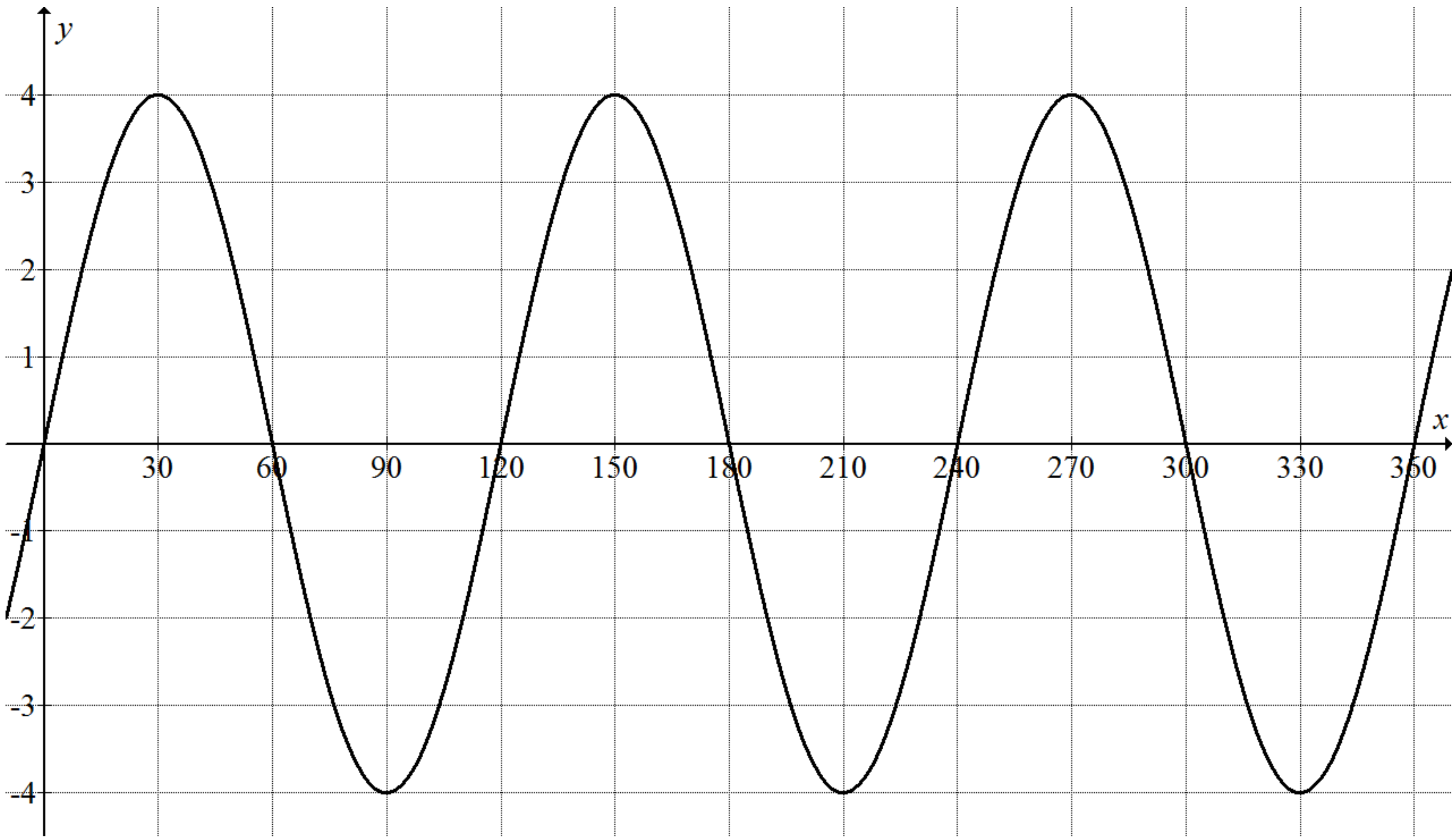
6. Sketch the graph of $y = 2\sin 3x°, 0 \leq x \leq 360$.

7. Sketch the graph of $y = \tan(x - 45)°, 0 \leq x \leq 360$.

8. Sketch the graph of $y = 5\sin 2x°, 0 \leq x \leq 360$.

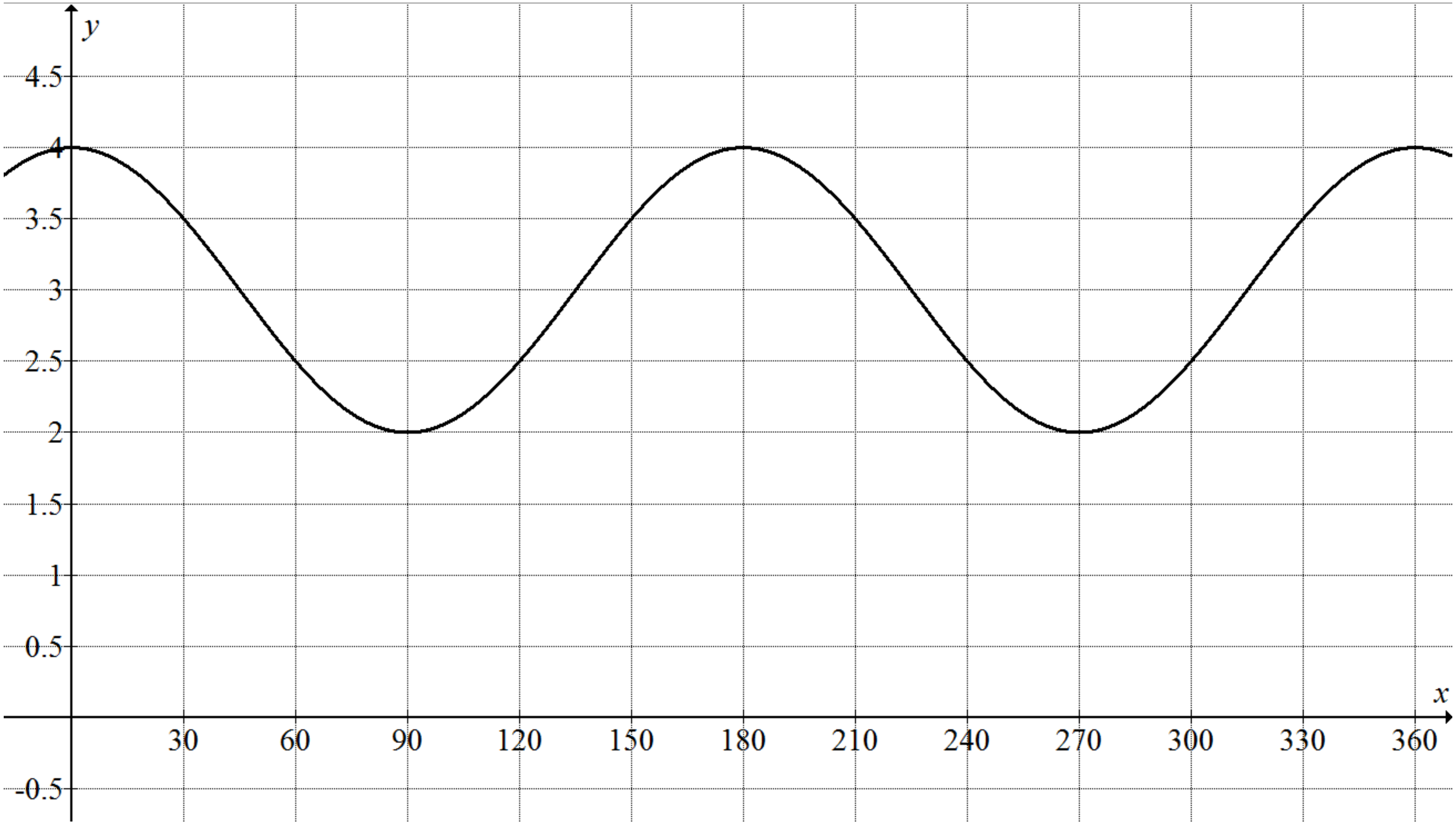
O4 I can identify Trig Graphs.

1. Part of the graph of $y = a\sin bx°$ is shown.



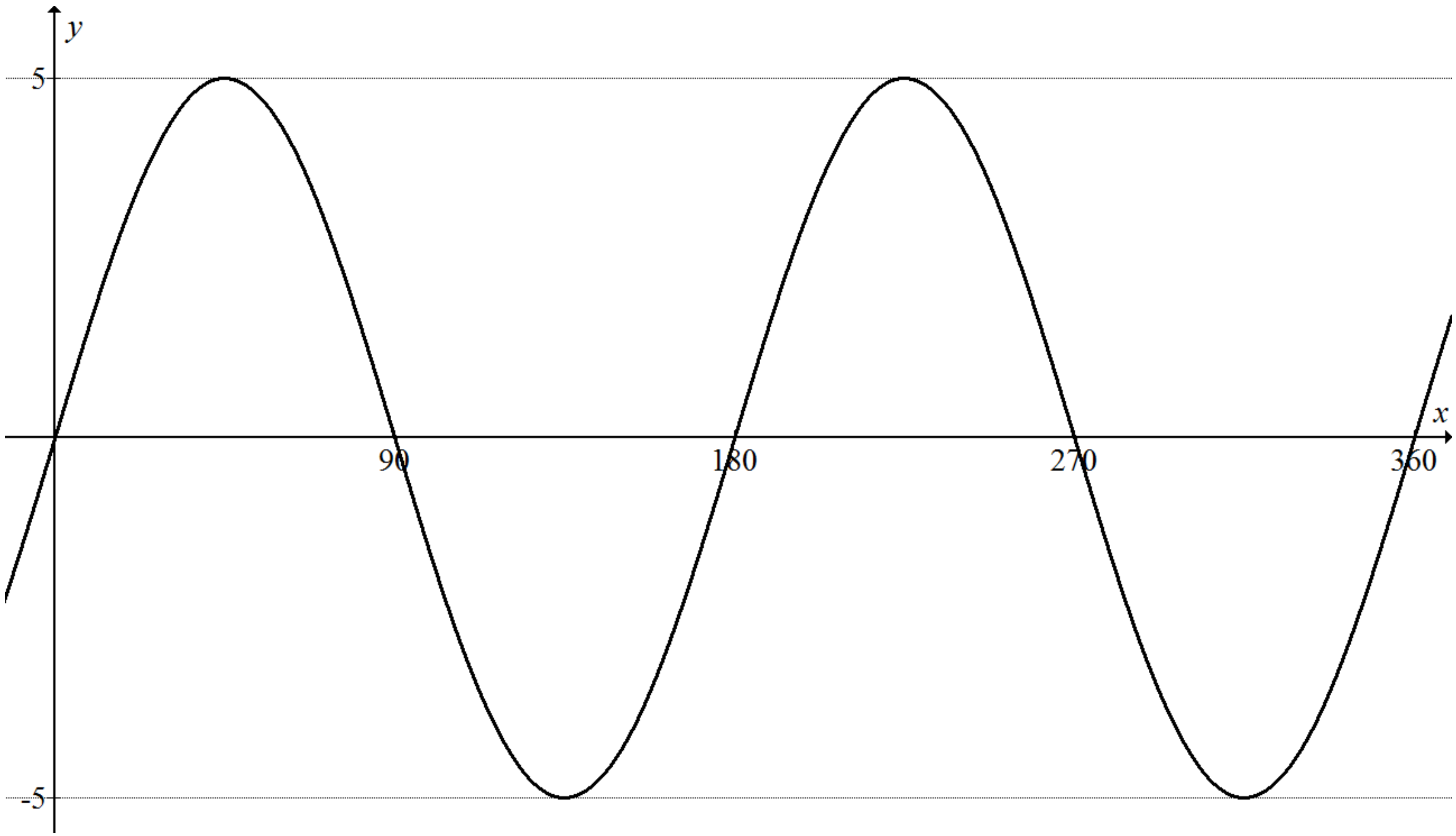
Write down the values of a and b.

2. Part of the graph of $y = \cos bx° + c$ is shown.



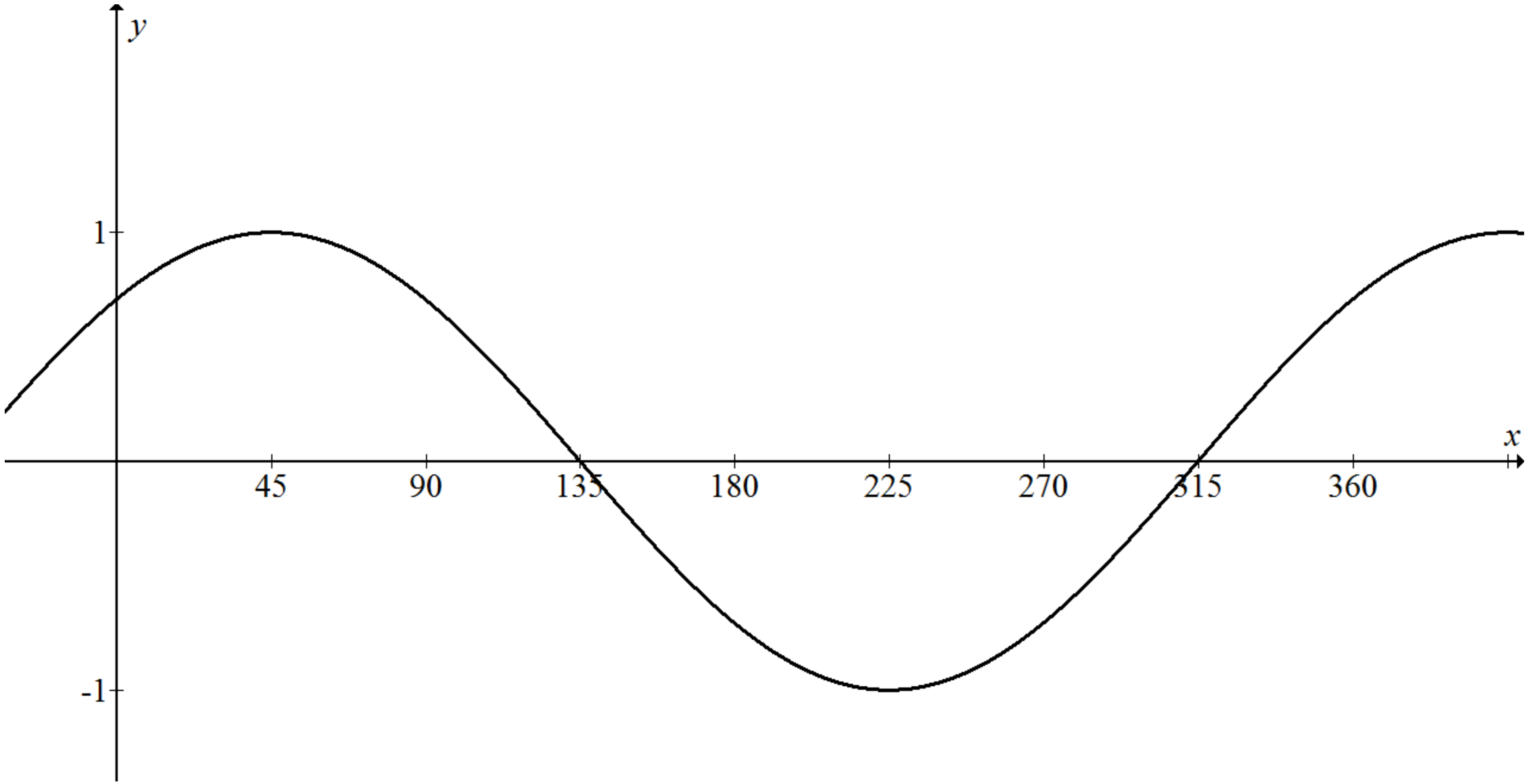
Write down the values of b and c.

3. Part of the graph of $y = a \sin bx°$ is shown.



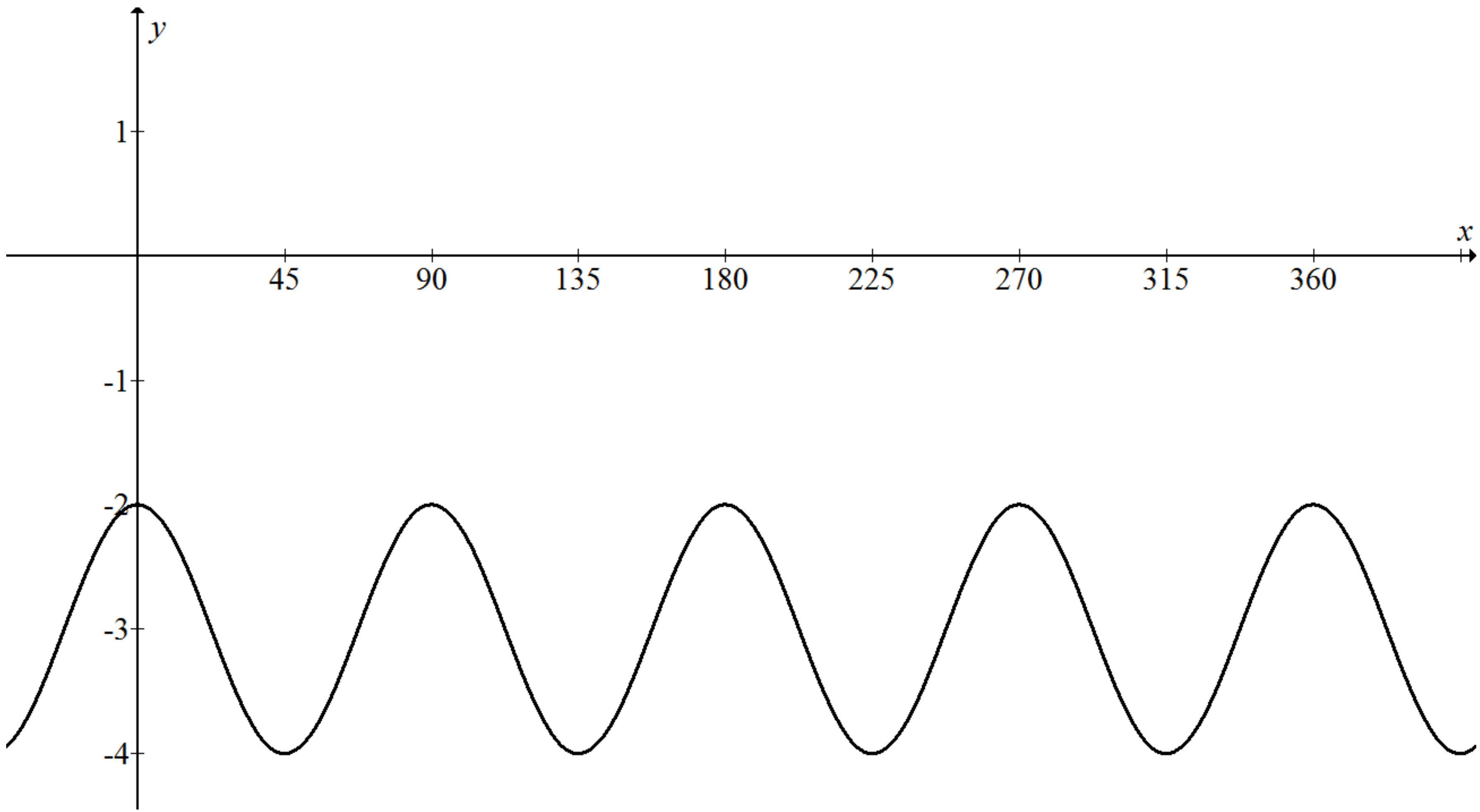
Write down the values of a and b.

4. Part of the graph of $y = \cos(x - a)°$ is shown.



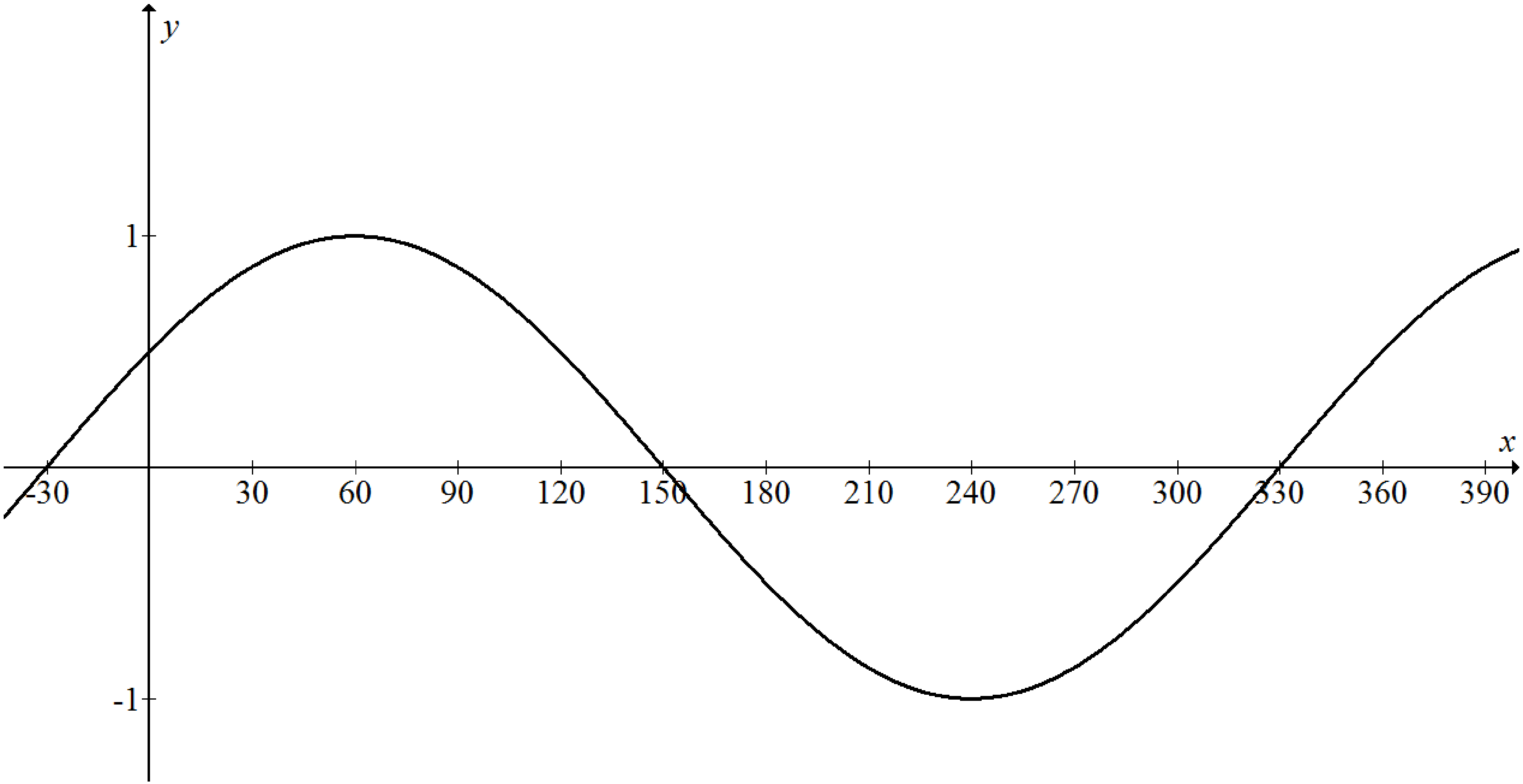
Write down the value of a.

5. Part of the graph of $y = \cos bx° + c$ is shown.



Write down the values of b and c.

6. Part of the graph of $y = \sin(x + a)°$ is shown.



Write down the value of a.

O5 Related Angles in the Four Quadrants

1. For all acute angles, $A°$, the following are true:

$\sin A° = \sin(180 - A)° = \sin(180 + A)° = \sin(360 - A)°$

Test this conjecture for

(a) $A = 30$ (b) $A = 45$ (c) $A = 60$

2. For all acute angles, $A°$, the following are true:

$\cos A° = -\cos(180 - A)° = -\cos(180 + A)° = \cos(360 - A)°$

Test this conjecture for

(a) $A = 15$ (b) $A = 75$ (c) $A = 30$

3. For all acute angles $(A°)$ then the following is true:

$\tan A° = \tan(180 - A)° = \tan(180 + A)° = \tan(360 - A)°$

Test this conjecture for

(a) $A = 15$ (b) $A = 30$ (c) $A = 75$

Make a note of this

Every acute angle $A°$ (between $0°$ and $90°$ which is the first quadrant) have related angles

$(180 - A)°$ in the second quadrant (between $90°$ and $180°$)

$(180 + A)°$ in the third quadrant (between $180°$ and $270°$)

$(360 - A)°$ in the fourth quadrant (between $270°$ and $360°$)

Make a note of this

For every acute angle $A°$ then:

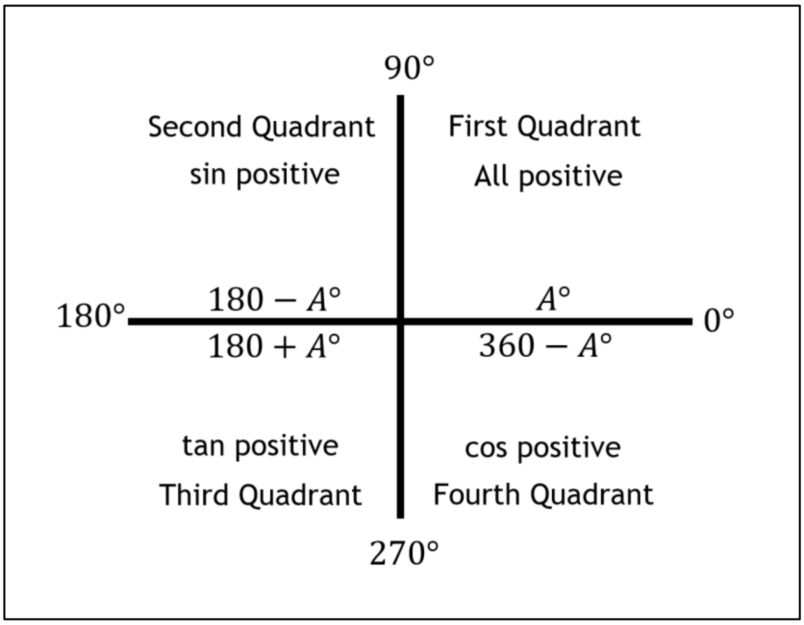
$\sin A°, \cos A°$ and $\tan A°$ will be positive

$\text{Sin}(180 - A)°$ will be positive and
$\cos(180 - A)°$ and $\tan(180 - A)°$ will be negative.

$\tan(180 + A)°$ will be positive and
$\sin(180 + A)°$ and $\cos(180 + A)°$ will be negative.

$\cos(360 - A)°$ will be positive and
$\sin(360 - A)°$ and $\tan(360 - A)°$ will be negative.

In summary



4. The answers to the following questions should be “true” or “false”.

(a) If $\cos 45^\circ = 0 \cdot 707$ then $\cos 315^\circ = 0 \cdot 707$.

(b) If $\sin 30^\circ = 0 \cdot 5$ then $\sin 150^\circ = -0 \cdot 5$.

(c) If $\tan 70^\circ = 2 \cdot 74$ then $\tan 110^\circ = 2 \cdot 74$.

(d) If $\cos 15^\circ = 0 \cdot 966$ then $\cos 195^\circ = -0 \cdot 966$.

(e) If $\sin 60^\circ = 0 \cdot 866$ then $\sin 300^\circ = 0 \cdot 5$.

(f) If $\cos 52^\circ = 0 \cdot 616$ then $\cos 232^\circ = 0 \cdot 616$.

5. The answers to the following questions should be “true” or “false”.

(a) If $\sin x^\circ$ is positive, $\cos x^\circ$ is negative and $\tan x^\circ$ is negative then x° is in the fourth quadrant.

(b) If $\sin x^\circ$ is negative, $\cos x^\circ$ is positive and $\tan x^\circ$ is negative then x° is in the third quadrant.

(c) If $\sin x^\circ$ is negative, $\cos x^\circ$ is negative and $\tan x^\circ$ is positive then x° is in the third quadrant.

(d) If $\sin x^\circ$ is positive, $\cos x^\circ$ is negative and $\tan x^\circ$ is negative then x° is in the second quadrant.

6. An angle, b°, can be described by the following statements.

- b° is greater than 0 and less than 360
- $\sin b^\circ$ is negative
- $\cos b^\circ$ is positive
- $\tan b^\circ$ is negative

Write down one possible value for b.

Book 21 - #2

DEADLINE

Cumulative Exercises - Brackets

1. (a) Multiply out the brackets and collect like terms

$$(2x - 3)(4x - 5)$$

(b) Multiply out the brackets and collect like terms

$$(3x + 2)(x^2 + 5x - 1)$$

2. Remove the brackets and simplify $(2x + 3)^2 - 3(x^2 - 6)$

3. Multiply out the brackets and collect like terms

$$(x + 5)(2x^2 - 3x - 1)$$

4. Expand and fully simplify $x(3x - 2)^2$

5. Given that $x^2 - 10x + 18 = (x - a)^2 + b$, find the values of a and b.

6. Given that $x^2 + 4x + 12 = (x + p)^2 + q$, find the values of p and q.

7. Given that $x^2 - 6x - 9 = (x - r)^2 + s$, find the values of r and s.

O6 I know the common exact values in the four quadrants.

1. Write down the exact value of:

(a) $\sin 30°$ (b) $\cos 30°$ (c) $\tan 30°$

(d) $\sin 60°$ (e) $\cos 60°$ (f) $\tan 60°$

(g) $\sin 45°$ (h) $\cos 45°$ (i) $\tan 45°$

(j) $\sin 0°$ (k) $\cos 0°$ (l) $\tan 0°$

(m) $\sin 90°$ (n) $\cos 90°$ (o) $\tan 90°$

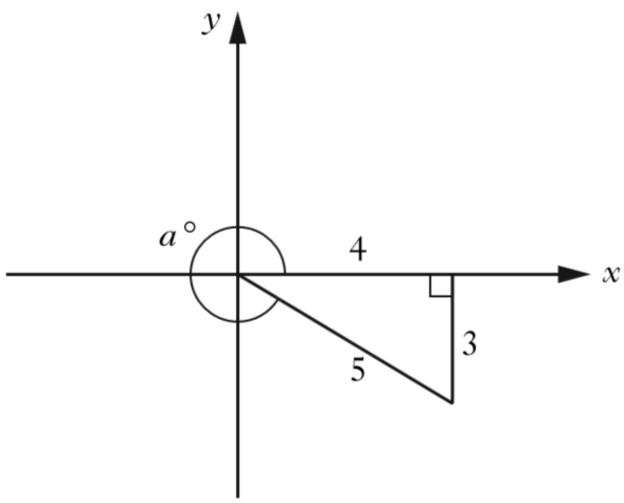
2. Write down the exact value of:

(a) $\sin 150°$ (b) $\cos 210°$ (c) $\tan 330°$

(d) $\sin 240°$ (e) $\cos 300°$ (f) $\tan 120°$

(g) $\sin 315°$ (h) $\cos 135°$ (i) $\tan 225°$

3. For the diagram shown, write down the exact value of $\cos a°$.



O7 I can solve Trig Equations.

1. Solve the following equations

(a) $2\sin x° + 3 = 4,$ $\quad 0 \leq x < 360.$

(b) $4\cos x° + 3 = 0,$ $\quad 0 \leq x \leq 360.$

(c) $7\sin x° + 1 = -5,$ $\quad 0 \leq x \leq 360.$

(d) $5\tan x° - 6 = 2,$ $\quad 0 \leq x < 360.$

2. Solve the following equations

(a) $2\tan x° - 3 = 5,$ $\quad 0 \leq x \leq 360.$

(b) $5\cos x° + 3 = 1,$ $\quad 0 \leq x \leq 360.$

(c) $4\tan x° + 8 = 5,$ $\quad 0 \leq x \leq 360.$

(d) $5\sin x° + 4 = 0,$ $\quad 0 \leq x < 360.$

3. Solve the equation

(a) $\tan 40° = 2\cos x° - 1,$ $\quad 0 \leq x < 360.$

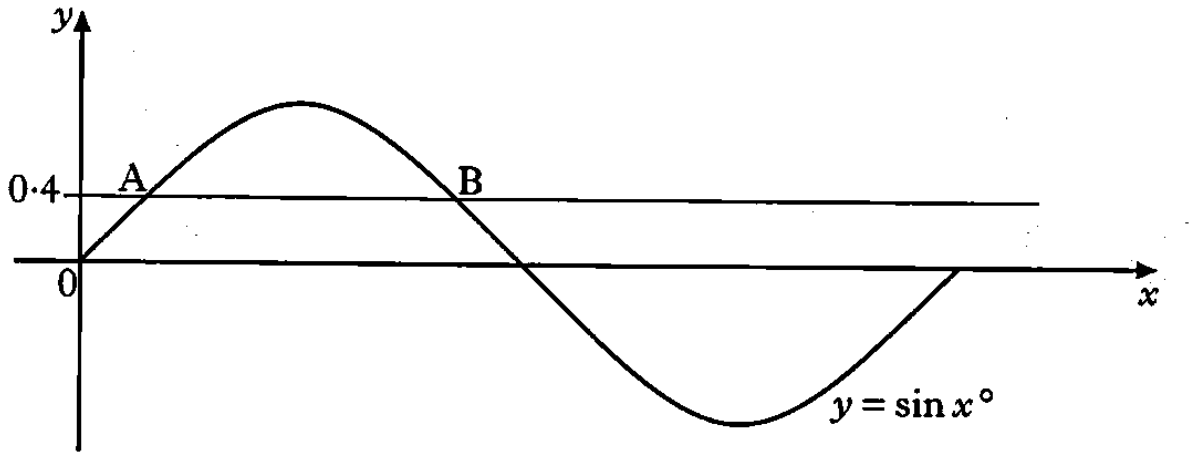
(b) $\sin 30° = 2\sin x° + \frac{3}{2},$ $\quad 0 \leq x \leq 360.$

(c) $\cos 60° = \tan x° + 5,$ $\quad 0 \leq x \leq 360.$

(d) $\tan 45° = 2\cos x° + 2,$ $\quad 0 \leq x < 360.$

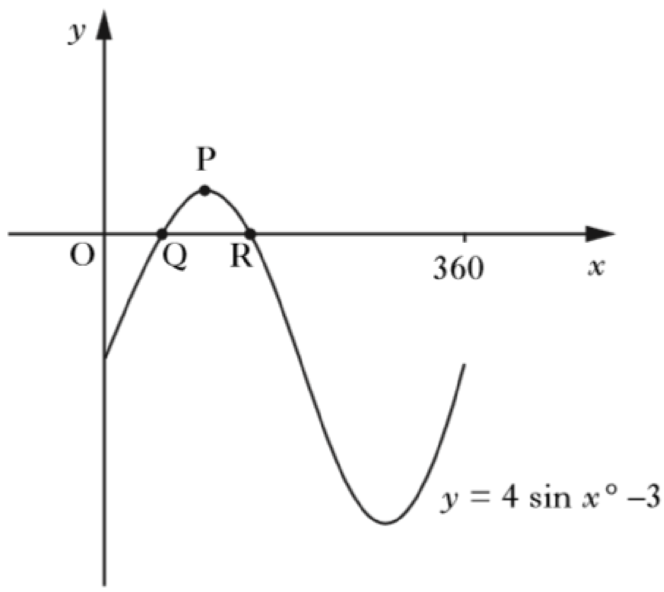
4. The diagram below shows part of the graph of $y = \sin x°$.

The line $y = 0 \cdot 4$ is drawn and cuts the graph of $y = \sin x°$ at A and B.



Find the x-coordinate of A and B.

5. Part of a graph of $y = 4\sin x° - 3$ is shown. The graph cuts the x-axis at Q and R. P is the maximum turning point.

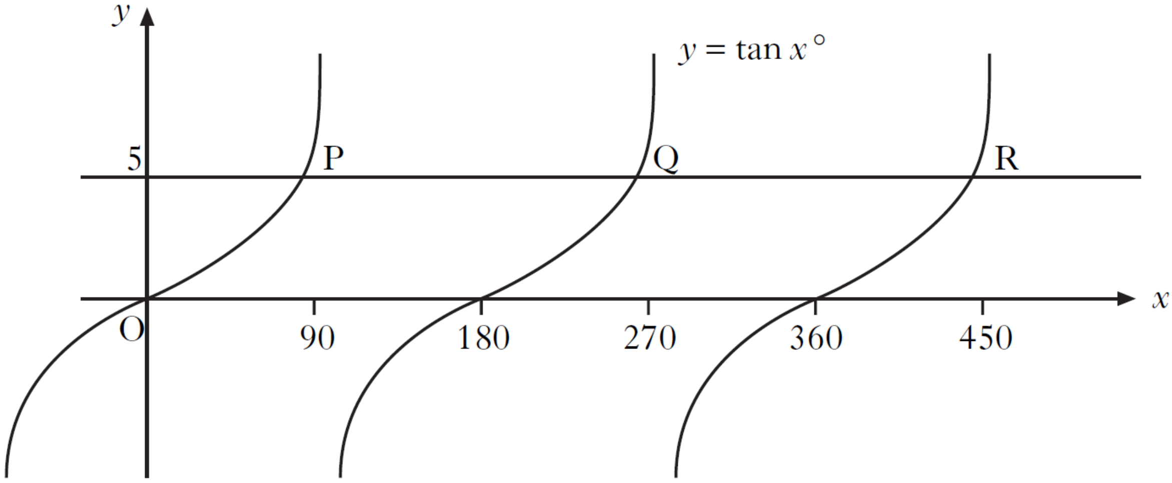


(a) Write down the coordinates of P.

(b) Calculate the x-coordinate of Q and R.

6. The diagram shows part of the graph of $y = \tan x°$.

The line $y = 5$ is drawn and intersects the graph of $y = \tan x°$ at P and Q.



(a) Find the x-coordinate of P and Q.

(b) Write down the x-coordinate of R, where the line $y = 5$ next intersects the graph of $y = \tan x°$.

7. If $f(x) = 3\sin x°,\ \ 0 \leq x \leq 360$

(a) Find $f(270)$.

(b) $f(t) = 0 \cdot 6$.

Find two possible values of t.

8. A function is defined $f(x) = 5\tan x°,\ \ 0 \leq x \leq 360$

If $f(a) = 15$, find two possible values of a.

9. The depth of water, D metres, in a harbour is given by the formula

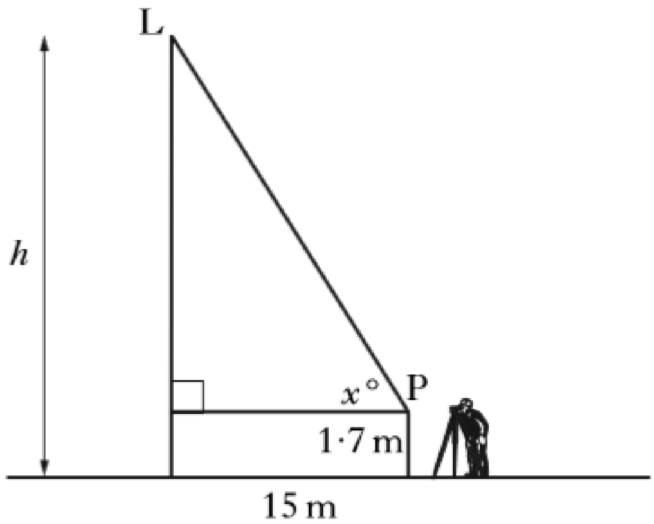
$$D = 3 + 1 \cdot 75 \sin 30h°$$

Where h is the number of hours after midnight.

(a) Calculate the depth of water at 5am.

(b) Calculate the maximum difference in depth of water in the harbour.

10. In the diagram below, the point L represents a lift.



The height, h metres, of the lift above the ground is given by the formula

$$h = 15\tan x° + 1 \cdot 7$$

Where $x°$ is the angle of elevation from the surveyor at point P.

(a) What is the height of the lift above the ground when the angle of elevation from P is $25°$?

(b) What is the angle of elevation at the point P when the height of the lift above the ground is 18·4 metres?

11. Cara goes on the London Eye.

Her height, h metres, above the ground is given by the formula

$$h = -59\cos t° + 61$$

where t is the number of seconds after the start.

(a) Calculate Cara's height above the ground 30 seconds after the start.

(b) When will Cara first reach a height of 80 metres above the ground?

(c) When will she next be at a height of 80 metres above the ground?

12. (a) Solve algebraically the equation

$$\sqrt{3}\sin x° - 1 = 0, \quad 0 \leq x < 360.$$

(b) Hence write down the solution of the equation

$$\sqrt{3}\sin 2x° - 1 = 0, \quad 0 \leq x < 90.$$

1. Evaluate

(a) $\frac{2}{3} \div 1\frac{1}{5}$ (b) $4\frac{1}{3} - 1\frac{1}{2}$ (c) $3\frac{1}{3} \times \frac{4}{5}$

2. There are 31·7 million vehicles in UK.

It is estimated that this number will increase at a rate of 4% each year.

If this estimate is correct, how many vehicles will there be in 3 years' time?

Give your answer correct to 3 significant figures.

3. A car is valued at £4200.

This is 16% less than last year's value.

What was the value of the car last year?

4. Simplify $\frac{3x-15}{(x-5)^2}$

5. (a) Express $\frac{1}{p} - \frac{2}{(p+5)}$, $p \neq 0,\ p \neq -5$ as a single fraction in its simplest form.

(b) Express $\frac{3}{x} - \frac{4}{x+1}$, $x \neq 0,\ x \neq -1$ as a single fraction in its simplest form.

O8 I can manipulate Trig Identities.

Resilience Required

You are unlikely to be able to successfully complete this exercise on your first attempt.

It may take you several attempts before you can do all these questions without checking your notes. So, resilience required!

1. If $\sin x^\circ = \frac{4}{5}$ and $\cos x^\circ = \frac{3}{5}$, calculate the value of $\tan x^\circ$.

2. Simplify $\tan x^\circ \cos x^\circ$

3. Simplify

(a) $\dfrac{\cos x^\circ \tan x^\circ}{\sin x^\circ}$ (b) $\dfrac{\cos^3 x^\circ}{1-\sin^2 x^\circ}$.

4. Prove that $\dfrac{\sin^2 A}{1-\sin^2 A} = \tan^2 A$

5. Show that $\dfrac{1}{\cos x^\circ} - \sin x^\circ \tan x^\circ = \cos x^\circ$

Book 21 - #3

DEADLINE