

EF 1.3 - Algebraic Fractions

Section A - Revision

This section will help you revise previous learning which is required in this topic.

R1 I have revised how to use the four operations applied to vulgar fractions and mixed numbers.

1. Carry out the following calculations, showing clearly all working, leaving your answer as a vulgar fraction.

(a) $\frac{2}{3} + \frac{1}{4}$

(b) $\frac{4}{5} \times \frac{1}{6}$

(c) $\frac{7}{8} - \frac{3}{5}$

(d) $\frac{9}{14} \div \frac{3}{7}$

(e) $\frac{3}{8} + \frac{1}{6}$

(f) $\frac{1}{12} \div \frac{3}{4}$

(g) $\frac{2}{15} \times \frac{5}{12}$

(h) $\frac{5}{12} + \frac{1}{3}$

(i) $\frac{4}{5} - \frac{3}{11}$

2. Carry out the following calculations, showing clearly all working, leaving your answer as a mixed.

(a) $\frac{4}{5} + \frac{3}{4}$

(b) $\frac{7}{11} + \frac{3}{5}$

(c) $\frac{3}{4} \div \frac{7}{8}$

(d) $\frac{5}{7} \div \frac{10}{21}$

(e) $\frac{7}{9} + \frac{4}{5}$

(f) $\frac{3}{4} \div \frac{1}{20}$

(g) $\frac{2}{5} + \frac{7}{8}$

(h) $\frac{7}{10} + \frac{5}{6}$

(i) $\frac{4}{5} \div \frac{3}{11}$

3. Carry out the following calculations, showing clearly all working, leaving your answer as a mixed number where appropriate.

(a) $3\frac{1}{2} + \frac{5}{6}$

(b) $2\frac{3}{10} \div \frac{1}{15}$

(c) $5\frac{1}{8} - \frac{4}{5}$

(d) $3\frac{1}{8} \times 1\frac{2}{5}$

(e) $4\frac{3}{4} - 2\frac{1}{6}$

(f) $2\frac{3}{10} \div 1\frac{3}{5}$

(g) $3\frac{4}{15} \times 1\frac{3}{7}$

(h) $4\frac{1}{15} + 3\frac{3}{5}$

(i) $7\frac{1}{5} - 1\frac{7}{11}$

Algebraic Fractions

R2 I have revised multiplying out brackets, gathering like terms and factorisation.

1. Multiply out the brackets and simplify

- | | | |
|-----------------------------|----------------------------|---------------------|
| (a) $3(2x + 5) + 6$ | (b) $5(x + 9) + 2x$ | (c) $7(3x - 2) + 4$ |
| (d) $4(2x + 7) + 5x$ | (e) $6(2x - 9) + 7x$ | (f) $6(2x - 9) + 7$ |
| (g) $2(3x + 1) + 3(x - 5)$ | (h) $5(x + 4) - 3(x - 7)$ | (i) $4 - 5(2x + 1)$ |
| (j) $4(5x + 2) - 7(2x + 3)$ | (k) $9(2x + 3) + 2(x + 4)$ | (l) $8 - 3(x - 9)$ |

2. Multiply out the brackets and simplify

- | | | |
|------------------------------|------------------------------|----------------------|
| (a) $(x + 3)(x + 4)$ | (b) $(x + 2)(x - 5)$ | (c) $(x - 7)(x + 3)$ |
| (d) $(x - 9)(x - 2)$ | (e) $(2x + 5)(x + 3)$ | (f) $(x - 1)(x + 4)$ |
| (g) $(3x - 2)(x - 9)$ | (h) $(5x + 1)(3x - 7)$ | (i) $(x - 3)(x - 5)$ |
| (j) $(x + 3)(x^2 - 3x + 4)$ | (k) $(x - 5)(2x^2 + 4x - 3)$ | (l) $(x - 2)(3 - x)$ |
| (m) $(2x + 5)(3x^2 - x - 8)$ | (n) $(3x - 1)(x^2 + x - 5)$ | (o) $(3 - x)(5 + x)$ |

3. Factorise completely

- | | | |
|---------------------|-----------------------|----------------------|
| (a) $8a + 12b$ | (b) $49 - p^2$ | (c) $x^2 + 5x + 6$ |
| (d) $2x^2 - 7x + 3$ | (e) $2xy + xz$ | (f) $x^2 - 25$ |
| (g) $9mn - 12mp$ | (h) $a^2 - 8a + 7$ | (i) $25g^2 - 49$ |
| (j) $9x^2 - 16$ | (k) $5x^2 + 4x - 1$ | (l) $15x^2 + 10x^2y$ |
| (m) $x^2 - 2x - 15$ | (n) $3x^2 + 2xy + 6x$ | (o) $6r^2 - 24q^2$ |
| (p) $8r^2 + 2r - 3$ | (q) $x^2 + 6x + 9$ | (r) $15 - 7r - 2r^2$ |

Algebraic Fractions

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test for Algebraic Fractions (Expressions and Formulae 1.3)

Practice Assessment Standard Questions

1. Write each algebraic fraction in its simplest form.

(a) $\frac{(2x-3)(x+1)}{(x+1)^2}, x \neq -1$

(b) $\frac{(3x-1)(x+2)}{(x+2)^2}, x \neq -2$

(c) $\frac{(x+3)(2x+1)}{(x+3)^2}, x \neq -3$

(d) $\frac{(x-1)(x+1)}{(x-1)^2}, x \neq 1$

(e) $\frac{(5x-2)(x+1)}{(x-3)(x+1)}, x \neq -1 \text{ or } 3$

(f) $\frac{(x-1)^2}{(x-1)(x+2)}, x \neq -2 \text{ or } 1$

(g) $\frac{(x+3)^2}{(x-2)(x+3)}, x \neq -3 \text{ or } 2$

(h) $\frac{(x-1)(x+8)}{(x-1)(x+1)}, x \neq -1 \text{ or } 1$

2. Write each of the following as a single fraction

(a) $\frac{2}{x} - \frac{3}{y}, x, y \neq 0$

(b) $\frac{x}{6} \times \frac{t}{y}, t, x, y \neq 0$

(c) $\frac{3}{x} + \frac{4}{y}, x, y \neq 0$

(d) $\frac{x}{8} \div \frac{t}{y}, t, x, y \neq 0$

(e) $\frac{3}{x} + \frac{5}{y}, x, y \neq 0$

(f) $\frac{1}{x} - \frac{1}{y}, x, y \neq 0$

(g) $\frac{7}{x} + \frac{3}{y}, x, y \neq 0$

(h) $\frac{x}{10} \times \frac{t}{y}, t, x, y \neq 0$

(i) $\frac{9}{x} - \frac{2}{y}, x, y \neq 0$

(j) $\frac{x}{5} \div \frac{t}{y}, t, x, y \neq 0$

(k) $\frac{x}{100} \times \frac{t}{y}, t, x, y \neq 0$

(l) $\frac{5}{x} - \frac{3}{y}, x, y \neq 0$

Algebraic Fractions

Section C - Operational Skills

This section provides problems with the operational skills associated with Algebraic fractions

N1 Simplifying Algebraic fractions

1. Simplify

(a) $\frac{6b^5}{6b^2}$

(b) $\frac{15x^6}{5x^4}$

(c) $\frac{14p^7}{21p^2}$

(d) $\frac{a^4b^7}{a^2b^3}$

(e) $\frac{x^4y^8}{xy^3}$

(f) $\frac{p^9q^8}{p^5q^4}$

(g) $\frac{5b^3}{5b^7}$

(h) $\frac{12x^3}{6x^8}$

(i) $\frac{9p}{15p^2}$

(j) $\frac{8a^4b}{4a^2b^3}$

(k) $\frac{6x^4y^5}{10xy^7}$

(l) $\frac{5p^3q^3}{10p^5q}$

2. Simplify

(a) $\frac{3(x-5)}{(x-5)^2}$

(b) $\frac{5(x+1)}{(x+1)(x-2)}$

(c) $\frac{(x+2)(x+3)}{x+2}$

(d) $\frac{(2x+5)^2}{(2x-1)(2x+5)}$

(e) $\frac{2x+8}{(x+4)^2}$

(f) $\frac{3x^2+6x}{x(3x-1)}$

(g) $\frac{2x^2+x-3}{3(2x+3)}$

(h) $\frac{x^2+4x+4}{(x-1)(x+2)}$

(i) $\frac{10x-5}{4x^2-4x+1}$

(j) $\frac{14x+21}{4x^2+12x+9}$

(k) $\frac{9x^2-1}{3x^2-7x+2}$

(l) $\frac{5x^2+4x-1}{25x^2-1}$

Algebraic Fractions

N2 Adding and subtracting algebraic Fractions

1. Write each of the following as a single fraction

(a) $\frac{a}{b} - \frac{b}{a}$ $a, b \neq 0$ (b) $\frac{x}{y} + \frac{y}{x}$ $x, y \neq 0$ (c) $\frac{3a}{3} + \frac{2}{a}$ $a \neq 0$

(d) $\frac{2a}{3b} - \frac{b}{2a}$ $a, b \neq 0$ (e) $\frac{x}{5y} + \frac{3y}{2x}$ $x, y \neq 0$ (f) $\frac{a}{4} + \frac{1}{a}$ $a \neq 0$

(g) $\frac{1}{b} - \frac{2}{ab}$ $a, b \neq 0$ (h) $\frac{3}{xy} + \frac{1}{x^2}$ $x, y \neq 0$ (i) $\frac{3}{a^2} + \frac{1}{a}$ $a \neq 0$

2. Express each of the following as a single fraction

(a) $\frac{1}{p} + \frac{2}{(p+5)}$ (b) $\frac{3}{x} - \frac{4}{x+1}$ (c) $\frac{2}{a} - \frac{3}{(a+4)}$

(d) $\frac{3}{(p+1)} - \frac{1}{(p+5)}$ (e) $\frac{2}{x-1} + \frac{4}{x+2}$ (f) $\frac{1}{(a-3)} + \frac{4}{(a+4)}$

(g) $\frac{p}{p-1} + \frac{1}{p}$ (h) $\frac{2}{x+2} - \frac{4x}{x+1}$ (i) $\frac{2a}{(a+2)} - \frac{5}{(a+4)}$

(j) $\frac{p+5}{2} + \frac{p}{3}$ (k) $\frac{x+1}{2} - \frac{x+3}{6}$ (l) $\frac{(a+3)}{2} - \frac{(2a-1)}{5}$

N3 Multiplying and dividing Algebraic fractions

1. Write each of the following as a single fraction

(a) $\frac{5p^2}{8} \div \frac{p}{2}$ (b) $\frac{s^2}{t} \times \frac{3t}{2s}$ (c) $\frac{x^5}{y} \times \frac{4y^3}{3x}$

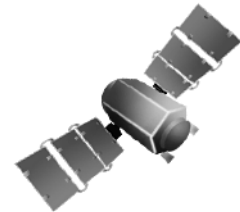
(d) $\frac{4p^2}{15} \div \frac{p}{3r^5}$ (e) $\frac{3s^2}{t^4} \times \frac{7t}{9s^5}$ (f) $\frac{3p^4}{49} \div \frac{12p}{14}$

Algebraic Fractions

Section D - Course Assessment Section

This section provides problems which you can use to practise course assessment style questions.

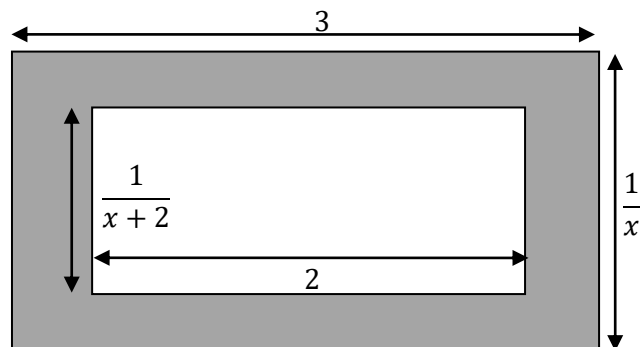
1. A satellite travels a distance of $3xy^3$ kilometres in $12x^2y$ hours. Calculate the speed of the satellite in kilometres per hour. **Give your answer as a fraction in its simplest form.**



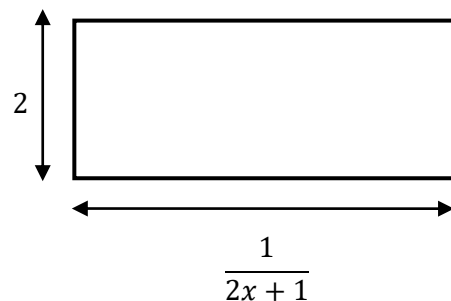
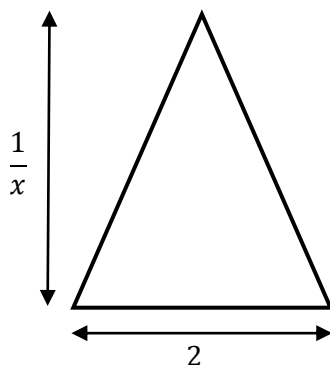
2. A particle travels at a speed of $\frac{5x^2}{2}$ metres per second for $\frac{4t^3}{3x}$ seconds. Calculate the distance travelled by the particle.

3. Look at the diagram.
Show that the shade area can be represented by

$$A = \frac{x+6}{x(x+2)}$$



4. In the diagram below, the triangle has an area 3 square units greater than the area of the rectangle.



Show that $\frac{1}{x(2x+1)} = 3$.

Algebraic Fractions

Answers

Section A

R1

- Q1 (a) $\frac{11}{12}$ (b) $\frac{2}{15}$ (c) $\frac{11}{40}$ (d) $\frac{3}{2}$ (e) $\frac{13}{24}$
(f) $\frac{1}{9}$ (g) $\frac{1}{18}$ (h) $\frac{3}{4}$ (i) $\frac{29}{55}$
- Q2 (a) $1\frac{11}{20}$ (b) $1\frac{13}{55}$ (c) $\frac{6}{7}$ (d) $1\frac{1}{2}$ (e) $1\frac{26}{45}$
(f) 15 (g) $1\frac{11}{40}$ (h) $1\frac{8}{15}$ (i) $2\frac{14}{15}$
- Q3 (a) $4\frac{1}{3}$ (b) $34\frac{1}{2}$ (c) $4\frac{13}{40}$ (d) $4\frac{3}{8}$ (e) $2\frac{7}{12}$
(f) $1\frac{7}{16}$ (g) $4\frac{2}{3}$ (h) $7\frac{2}{3}$ (i) $5\frac{31}{55}$

R2

- Q1 (a) $6x + 21$ (b) $7x + 45$ (c) $21x - 10$ (d) $13x + 28$ (e) $19x - 54$
(f) $12x - 47$ (g) $9x - 13$ (h) $2x + 41$ (i) $-1 - 10x$ (j) $6x - 13$
(k) $20x + 35$ (l) $35 - 3x$
- Q2 (a) $x^2 + 7x + 12$ (b) $x^2 - 3x - 10$
(c) $x^2 - 4x - 21$ (d) $x^2 - 11x + 18$
(e) $2x^2 + 11x + 15$ (f) $x^2 + 3x - 4$
(g) $3x^2 - 29x + 18$ (h) $15x^2 - 32x - 7$
(i) $x^2 - 8x + 15$ (j) $x^3 - 5x + 12$
(k) $2x^3 - 6x^2 - 23x + 15$ (l) $-x^2 + 5x - 6$
(m) $6x^3 + 13x^2 - 21x - 40$ (n) $3x^3 + 2x^2 - 16x + 5$
(o) $-x^2 - 2x + 15$
- Q3 (a) $4(2a + 3b)$ (b) $(7 - p)(7 + p)$
(c) $(x + 2)(x + 3)$ (d) $(2x - 1)(x - 3)$
(e) $x(2y + z)$ (f) $(x - 5)(x + 5)$

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(g) $3m(3n - 4p)$

(h) $(a - 7)(a - 1)$

(i) $(5g - 7)(5g + 7)$

(j) $(3x - 4)(3x + 4)$

(k) $(5x - 1)(x + 1)$

(l) $5x^2(3 + 2y)$

(m) $(x - 5)(x + 3)$

(n) $x(3x + 2y + 6)$

(o) $6(r - 2q)(r + 2q)$

(p) $(2r - 1)(4r + 3)$

(q) $(x + 3)^2$

(r) $(5 + r)(3 - 2r)$

Section B

Q1 (a) $\frac{2x-3}{x+1}$ (b) $\frac{3x-1}{x+2}$ (c) $\frac{2x+1}{x+3}$ (d) $\frac{x+1}{x-1}$ (e) $\frac{5x-2}{x-3}$

(f) $\frac{x-1}{x+2}$ (g) $\frac{x+3}{x-2}$ (h) $\frac{x+8}{x+1}$

Q2 (a) $\frac{2y-3x}{xy}$ (b) $\frac{tx}{6y}$ (c) $\frac{3y+4x}{xy}$ (d) $\frac{xy}{8t}$ (e) $\frac{3y+5x}{xy}$

(f) $\frac{y-x}{xy}$ (g) $\frac{7y+3x}{xy}$ (h) $\frac{xt}{10y}$ (i) $\frac{9y-2x}{xy}$ (j) $\frac{xy}{5t}$

(k) $\frac{tx}{100y}$ (l) $\frac{5y-3x}{xy}$

Section C

N1

Q1 (a) b^3 (b) $3x^2$ (c) $\frac{2p^5}{3}$ (d) a^2b^4 (e) x^3y^5

(f) p^4q^4 (g) $\frac{1}{b^4}$ (h) $\frac{2}{x^5}$ (i) $\frac{3}{5p}$ (j) $\frac{2a^2}{b^2}$

(k) $\frac{3x^3}{5y^2}$ (l) $\frac{q^2}{2p^2}$

Q2 (a) $\frac{3}{x-5}$ (b) $\frac{5}{x-2}$ (c) $x + 3$ (d) $\frac{2x+5}{2x-1}$ (e) $\frac{2}{x+4}$

(f) $\frac{3(x+2)}{(3x-1)}$ (g) $\frac{x-1}{3}$ (h) $\frac{x+2}{x-1}$ (i) $\frac{5}{2x-1}$ (j) $\frac{7}{2x+3}$

(k) $\frac{3x+1}{x-2}$ (l) $\frac{x+1}{5x+1}$

N2

Algebraic Fractions

Q1 (a) $\frac{a^2-b^2}{ab}$ (b) $\frac{x^2+y^2}{xy}$ (c) $\frac{a^2+2}{a}$ (d) $\frac{4a^2-3b^2}{6ab}$ (e) $\frac{2x^2+15y^2}{10xy}$
(f) $\frac{a^2+4}{4a}$ (g) $\frac{a-2}{ab}$ (h) $\frac{3x+y}{x^2y}$ (i) $\frac{3+a}{a^2}$

Q2 (a) $\frac{3p+5}{p(p+5)}$ (b) $\frac{3-x}{x(x+1)}$ (c) $\frac{8-a}{a(a+4)}$ (d) $\frac{2p+14}{(p+1)(p+5)}$ (e) $\frac{6x}{(x-1)(x+2)}$
(f) $\frac{5a-8}{(a-3)(a+4)}$ (g) $\frac{p^2+p-1}{p(p-1)}$ (h) $\frac{-4x^2-6x+2}{(x+2)(x+1)}$ (i) $\frac{2a^2+3a-10}{(a+2)(a+4)}$ (j) $\frac{5p+15}{6}$
(k) $\frac{x}{3}$ (l) $\frac{a+17}{10}$

N3

Q1 (a) $\frac{5p}{4}$ (b) $\frac{3s}{2}$ (c) $\frac{4x^4y^2}{3}$ (d) $\frac{4pr^5}{5}$ (e) $\frac{7}{3s^3t^3}$
(f) $\frac{p^3}{14}$

Section D

Q1 $S = \frac{y^2}{4x}$ km/h

Q2 $D = \frac{10xt^3}{3}$ metres

Q3 $A = \frac{x+6}{x(x+2)}$ as required.

Q4 $\frac{1}{x(2x+1)}$ as required.